

HOMEWORK 6  
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. Let  $A(x, y)$  be the conditional sentence

If  $x = 3$  and  $y = 5$ , then  $xy = 15$ .

- (a) Use one of the generalized De Morgan's laws to write the negation of the sentence  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) A(x, y)$ .
- (b) Is  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) A(x, y)$  true or false? Explain your answer.
- (c) Let  $B(x, y)$  be the converse of  $A(x, y)$ . Is  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) B(x, y)$  true or false? Explain your answer.
2. **[Falkner Section 3 Exercise 7]** Let  $P$  be the sentence

$(\exists x \in \mathbb{R})(x \geq 0 \text{ and } \sqrt{x+2} < \sqrt{x} + \sqrt{2})$ .

- (a) Use one of the generalized De Morgan's laws and one of the ordinary De Morgan's laws to show that  $\neg P$  is logically equivalent to
- $(\forall x \in \mathbb{R})(x < 0 \text{ or } \sqrt{x+2} \geq \sqrt{x} + \sqrt{2})$ .
- (b) Is  $P$  true or false? Provide a proof for your answer.
3. **[Falkner Section 3 Exercise 10]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your statement.

- (a)  $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = x)$ .
- (b)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = x)$ .
- (c)  $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = 0)$ .
- (d)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = 0)$ .
- (e)  $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(xy = 1)$ .
- (f)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy = 1)$ .

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 3 Exercise 9]** See book for problem statement.