

HOMEWORK 8
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 4 Exercise 1]** Let x , y , and z be integers.
 - (a) Prove that if x is even and y is even, then $x + y$ is even.
 - (b) Prove that if x is even and y is odd, then $x + y$ is odd.
 - (c) Suppose x , y , and z are odd. Is $x + y + z$ odd? Or is $x + y + z$ even? Explain your answer. You should not have to use the definitions of odd and even. Instead, you should be able to answer this part by combining one of parts (a) and (b) with Example 4.5.

2. **[Falkner Section 4 Exercise 2]** Let x , y , and z be integers.
 - (a) Prove that if x is odd and y is odd, then xy is odd.
 - (b) Suppose x , y , and z are odd. Is xyz odd? Or is xyz even? You should not have to use the definitions of odd and even. Instead, you should be able to answer this part by applying part (a).

3. **[Integers Exercise 1]** Prove Lemma 4: For any $a \in \mathbb{Z}$, $-(-a) = a$.

You may **only** assume the axioms for the integers and Lemmas 1–3 on the Integers handout. You may find the Additive Inverses axiom and Lemma 2 (Uniqueness of Additive Inverses) particularly useful.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 4 Exercise 3]** Let x be an integer. Prove that $x(x + 1)$ is even.
2. Let $r \neq 1$ be a real number. Prove that for every $n \in \mathbb{N}$,

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{r^n - 1}{r - 1}.$$

This is the (finite) **geometric series formula**.