Warm-Up:

- Let $a, b, c \in \mathbb{Z}$. Prove that if $\gcd(b,c) = 1$, then $\gcd(a, bc) = \gcd(a,b) \cdot \gcd(a,c)$.

- Compute $\gcd(a, bc)$ for (see Lecture 25 notes for proof)

  - $a = 18$, $b = 20$, $c = 21$
  - $a = 18$, $b = 10$, $c = 42$

Another application of FTA: Proving a root is irrational.

Ex: $\sqrt{6} \notin \mathbb{Q}$.

Proof: Assume, to get a contradiction, that $\sqrt{6} \in \mathbb{Q}$. Then $\sqrt{6} = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$ with $b \neq 0$.

Thus, $6 = \frac{a^2}{b^2}$, or $a^2 = 6b^2$.

Now, look at the prime factorizations on each side. (**Note**: $6b^2 \geq 6 \geq 2$)

In the factorization of a^2, every prime must occur an even number of times.

In the factorization of $6b^2$, the primes 2 and 3 each occur an odd number of times, contradicting uniqueness of prime factorization. ■

Alternate proof: Add the assumption that $\frac{a}{b}$ is in lowest terms. That is, a and b are relatively prime.

Then any prime dividing b also divides a^2, hence divides a. (why?)

But $\gcd(a, b) = 1$, so this is only possible if $b = 1$.

Thus, $\sqrt{6} = \frac{a}{1} = a \in \mathbb{Z}$, a contradiction. ■

Sets

"**Def**": A **set** is an unordered collection of objects, called **elements** of the set.

Actual definition is a list of axioms

One way to describe a set: list its elements inside braces.

Ex: $\{1,2,3\}$, {red, blue}, {☺, $, ★, □} are sets

Important notes:

- The elements in a set are **unordered**.

  So

  $$\{1,2,3\},\ \{1,3,2\},\ \{2,1,3\},\ \{2,3,1\},\ \{3,1,2\},\ \{3,2,1\}$$

  are six ways of writing **the same set**.

- The elements are **distinct** – no object can appear more than once. If we write

  $$\{1,1,1,2,2,2,2,2,2,3,3\},$$

  this means the set $\{1,2,3\}$.

If A is a set, then $x \in A$ means x is an element of A. $x \notin A$ means x is not an element of A.

<u>Ex</u>: $A = \{1, 2, 3\}$. Then $2 \in A$ and ☺ $\notin A$.

<u>Def</u>: The <u>empty set</u> is the set with no elements. It is denoted $\emptyset$.

$x \in \emptyset$ is <u>false</u> for every x.

Sets can have sets as elements.

<u>Ex</u>: $A = \{ \{1,2\}, \{\text{red}, \text{blue}\}, \$ \}$ is a set with three elements, two of which are sets themselves.

$$\{1,2\} \in A$$

$$1 \notin A$$