HW:
- neat/legible
- staple or paperclip if necessary
- complete sentences
- audience is your classmates
- collaborate responsibly

Warm-Up: For which real numbers x are the following sentences true?

(a) $(x > 2) \wedge (x < 5)$

(b) $\neg (x > 2)$

(c) $\neg \left[(x > 2) \wedge (x < 5) \right]$

How do the operations $\neg$, $\wedge$, $\vee$ interact with one another?

Warm-up part (c) gives one example.

<u>Thm</u> (DeMorgan's Laws) Let P and Q be sentences. Then

(a) $\neg(P \wedge Q)$ is logically equivalent to $\neg P \vee \neg Q$

(b) $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$

<u>Proof of (a):</u>

By truth table:

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

So we see $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$.

In words:

We wish to show $\neg(P \wedge Q)$ always has the same truth value as $\neg P \vee \neg Q$.

First, suppose $\neg(P \wedge Q)$ is true. Then $P \wedge Q$ is false, so at least one of P or Q is false.

But this means at least one of $\neg P$ or $\neg Q$ is true, so $\neg P \vee \neg Q$ is true.

Next, suppose $\neg(P \wedge Q)$ is false. Then $P \wedge Q$ is true, so both P and Q are true.

Now, both $\neg P$ and $\neg Q$ will be false, meaning $\neg P \vee \neg Q$ is false as well.

(b) HW 1

Thm (Distributive Laws)

Let P, Q, R be sentences. Then

(a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

(b) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Ex: P = "It is a nice day"
Q = "I will go for a walk"
R = "I will eat outside"

Proof of (b): We want to show these two sentences always have the same truth value.

First, suppose $P \vee (Q \wedge R)$ is true. Then either

- P is true

or

- $Q \wedge R$ is true, which means both Q and R are true

(or both).

In either case, $P \vee Q$ is true and $P \vee R$ is true, so $(P \vee Q) \wedge (P \vee R)$ is true.

The other possibility is that $P \vee (Q \wedge R)$ is false.
This means that <u>both</u>

- P is false

and

- $Q \wedge R$ is false, which in turn means at least one of Q or R is false.

But then at least one of $P \vee Q$ or $P \vee R$ is false.
So $(P \vee Q) \wedge (P \vee R)$ is false. ∎

As a truth table:

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F