Warm-Up: Let $f: \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$ be given by

$$f(m,n) = m - n.$$

e.g. $f(1,3) = -2$, $f(3,1) = 2$.

Show that $\mathrm{Rng}(f) = \mathbb{Z}$.

Ex: Let S be any set. Define a function

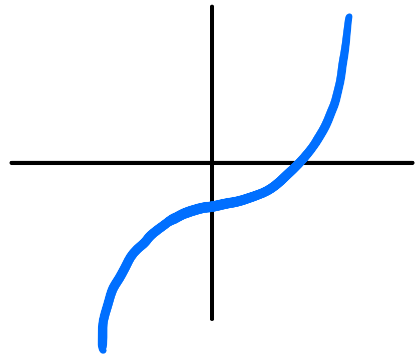
$$id_S : S \to S$$

by $id_S(x) = x$ for all $x \in S$.

This is called the identity function on S.

Graphs

Ex: For $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3 - 2$, the graph of f is



What is this? It's

$$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = x^3 - 2\}.$$

Def: Let $f: A \to B$ be a function. The graph of f is

$$\mathrm{Graph}(f) = \{(x,y) \in A \times B \mid y = f(x)\}.$$

Observe: For each $x \in A$, there is a unique $y \in B$ such that $(x,y) \in \mathrm{Graph}(f)$, namely $y = f(x)$.
"vertical line test"

Def: Let A and B be sets. A <u>function</u> $f: A \to B$ is a subset

$$\mathrm{Graph}(f) \subseteq A \times B$$

with the property that for all $x \in A$, there exists a unique $y \in B$ such that $(x,y) \in \mathrm{Graph}(f)$.

If $(x,y) \in \mathrm{Graph}(f)$, write $f(x) = y$.

<u>Note</u>:
- You don't have to use this definition. But it's more concrete than defining a function as a "rule".
- We can't always draw $\mathrm{Graph}(f)$.
- $\mathrm{Rng}(f) = \{ y \in B \mid (x,y) \in \mathrm{Graph}(f) \}$.