Warm-Up: Let P, Q be sentences.

Find a sentence using only the logical connectives $\neg$ and $\wedge$ which is logically equivalent to $P \vee Q$.

Another logical connective:

(4) **Implication**: $\Rightarrow$ means "implies" or "if-then"

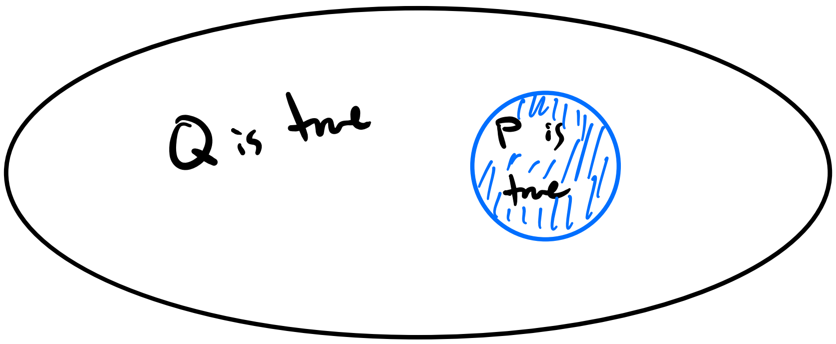
$P \Rightarrow Q$ means "if P is true, then Q is true"

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Why do the last 2 rows make sense?

Another perspective:

$P \Rightarrow Q$ is true when Q is "at least as true" as P.



Ex: If it's raining, then the ground is wet. T

If $x = 3$, then $x^2 = 9$. T

If $x^2 = 9$, then $x = 3$. F

If $0 > 1$, then $3^2 = 9$. T

If $0 > 1$, then the sun will explode today at 5 pm. T

Note:
- If P is false, then $P \Rightarrow Q$ is true.
- If Q is true, then $P \Rightarrow Q$ is true.

In fact,

Prop: Let P and Q be sentences. Then

$$P \Rightarrow Q \equiv \neg P \vee Q$$

Proof: The only situation in which $P \Rightarrow Q$ is false is if P is true and Q is false.

This is precisely when $\neg P \vee Q$ is false.

In all other cases, both $P \Rightarrow Q$ and $\neg P \vee Q$ are true.

Alternatively,

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

<u>Cor</u>: $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$.

<u>Proof</u>: $\neg(P \Rightarrow Q) \equiv \neg(\neg P \vee Q) \equiv \neg(\neg P) \wedge \neg Q$
$\equiv P \wedge \neg Q$ ∎

<u>Ex</u>: The negation of

"If it is Monday, then I will attend class"

is logically equivalent to

"It is Monday and I will not attend class."

A sentence of the form $P \Rightarrow Q$ is called a <u>conditional sentence</u>.

Ways to say $P \Rightarrow Q$:

"P implies Q"
"If P, then Q"
"P is sufficient for Q"
"Q is necessary for P"

<u>Ex</u>: I want

"If $x \geq 4$, then $x > 0$"

to be a true sentence. Is it?

Let's look at all possible x values.

	$x \geq 4$	$x > 0$	$(x \geq 4) \Rightarrow (x > 0)$
$x \geq 4$	T	T	T
$0 < x < 4$	F	T	T
$x \leq 0$	F	F	T

✓

Converse and Contrapositive

Let P and Q be sentences.

The contrapositive of $P \Rightarrow Q$ is the sentence

$$\neg Q \Rightarrow \neg P.$$

This is logically equivalent to $P \Rightarrow Q$.

The converse of $P \Rightarrow Q$ is the sentence

$$Q \Rightarrow P$$

This is NOT logically equivalent to $P \Rightarrow Q$.

Ex: "If it is raining, then the ground is wet."

Contrapositive:

"If the ground is dry, then it is not raining."

Converse:

"If the ground is wet, then it is raining."

Prop: $P \Rightarrow Q$ is logically equivalent to the contrapositive $\neg Q \Rightarrow \neg P$.

Proof:

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

We can see $P \Rightarrow Q$ is **NOT** logically equivalent to the converse $Q \Rightarrow P$ in the following truth table:

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T