Warm-Up: For each sentence, draw a number line and indicate all x-values making the sentence true.

(a) $(x>2) \wedge (x^2>4)$

(b) $(x>2) \vee (x^2>4)$

(c) $(x>2) \Rightarrow (x^2>4)$

(d) $(x>2) \Leftrightarrow (x^2>4)$

Quantifiers

The universal quantifier is $\forall$, which means "for all".

If $P(x)$ is a sentence involving the variable x, then $(\forall x)\, P(x)$ is the sentence

"For all x, $P(x)$"

also read

"For each x, $P(x)$"
"For all x, $P(x)$"
"For any x, $P(x)$"

The <u>existential quantifier</u> is $\exists$, which means "there exists".

$(\exists x)\, P(x)$ is the sentence

"there exists x such that $P(x)$"

also read

"for at least one x, $P(x)$"
"for some x, $P(x)$"

<u>Ex</u>: Which statements are true?

① $(\exists x \in \mathbb{R})\,(x+4=9)$

True, $x=5$

② $(\exists x \in \mathbb{R})\,[(x+4=9) \wedge (x \neq 5)]$

False, $x+4=9 \Rightarrow x=9-4=5$

③ $(\forall x \in \mathbb{R})\,(x+4=9)$

False, try $x=0$ or...

④ $(\exists x \in \mathbb{R})(x^2 + 6x + 8 \geq 0)$

True, try $x=0$.

⑤ $(\forall x \in \mathbb{R})(x^2 + 6x + 8 \geq 0)$

Can guess and check, or complete the square:

$$x^2 + 6x + 8 = x^2 + 6x + 9 - 1$$
$$= (x+3)^2 - 1$$

False, try $x = -3$

⑥ $(\forall x \in \mathbb{R})(x^2 + 6x + 10 \geq 0)$

True, since $x^2 + 6x + 10 = (x+3)^2 + 1 \geq 1 > 0$.

<u>Observation</u>:
- A single example proves a $\exists$ statement
- A single counterexample disproves a $\forall$ statement
- To prove a $\forall$ statement or disprove a $\exists$ statement, we need an argument that applies to <u>all</u> cases.