

**HOMEWORK 12**  
MATH 3345 – SPRING 2023 – KUTLER

**Exercises**

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. (a) Let  $n$  be an integer with  $n > 1$ . Prove that  $n$  is prime if and only if for every prime  $p$  such that  $p^2 \leq n$ ,  $p$  does not divide  $n$ .  
(b) Use part (a) to prove that 29 is prime.  
(c) Use part (a) to prove that 101 is prime.
2. Let  $a, b, q, r \in \mathbb{Z}$  such that  $a = bq + r$ .  
(a) Let  $d \in \mathbb{N}$ . Prove that  $d$  is a common divisor of  $a$  and  $b$  if and only if  $d$  is a common divisor of  $b$  and  $r$ .  
(b) Use part (a) to conclude that  $\gcd(a, b) = \gcd(b, r)$ .

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Let  $n$  be an integer. Prove that if  $3|n^2$ , then  $3|n$ .
2. Find an integer  $n$  such that  $4|n^2$  but  $4 \nmid n$ .