HOMEWORK 13
MATH 3345 – SPRING 2023 – KUTLER

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. (a) Let $x \in \mathbb{Z}$ and let p be a prime number. Prove that
$$\gcd(p, x) = \begin{cases} p & \text{if } p|x; \\ 1 & \text{if } p \nmid x. \end{cases}$$

   (b) Let p and q be distinct prime numbers (i.e., $p \neq q$). Use part (a) to show that $\gcd(p, q) = 1$.

2. Use the Euclidean algorithm to compute the following.

   (a) $\gcd(36, 22)$
   (b) $\gcd(96, 112)$
   (c) $\gcd(162, 31)$
   (d) $\gcd(-15, 45)$

3. (a) Use the Euclidean algorithm to compute $\gcd(350, 168)$.
   (b) Find integers x_1 and y_1 such that $350x_1 + 168y_1 = 14$.
   (c) Find integers x_2 and y_2 such that $350x_2 + 168y_2 = 42$. [HINT: $42 = 3 \cdot 14$.]
   (d) Prove that there do not exist integers x and y such that $350x + 168y = 15$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Let $a \in \mathbb{Z}$. Prove that

   (a) $\gcd(a, 1) = 1$.
   (b) $\gcd(a, 0) = |a|$.

2. Let $a, b \in \mathbb{Z}$ be integers which are not both zero. Prove that $\gcd(a, b) = \gcd(|a|, |b|)$.

3. Let $a, b \in \mathbb{Z}$ with $b \neq 0$. Prove that the fraction $\frac{a}{b}$ is in lowest terms if and only if $\gcd(a, b) = 1$.