HOMEWORK 17
MATH 3345 – SPRING 2023 – KUTLER

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. [**Falkner Section 8 Exercise 9**] Let $a, b \in \mathbb{R}$. Suppose $a \geq 0$ and $b \geq 0$. Prove that:

   (a) If $a < b$, then $a^2 < b^2$. [Use basic properties of inequalities.]

   (b) If $a^2 \leq b^2$, then $a \leq b$. [Do not use square roots. Use part (a).]

   (c) If $a^2 < b^2$, then $a < b$. [Again, do not use square roots. Use part (b).]

   (d) If $a < b$, then $\sqrt{a} < \sqrt{b}$. [Use part (c).]

2. [**Falkner Section 10 Exercise 1**] Which of the sets A, B, C, D, and E below are the same?

$$A = \{3\}, \qquad B = \{2, 4\}, \qquad C = \{x \mid x \text{ is prime}, x \text{ is odd, and } x < 5\},$$
$$D = \{x - 1 \mid x \text{ is prime}, x \text{ is odd, and } x < 5\}, \qquad E = \{x^2 + 2 \mid x \in \{-1, 1\}\}.$$

   Also, how many distinct sets are named here?

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. [**Falkner Section 8 Exercise 11**] Let $a, b \in \mathbb{R}$. Suppose $a > 0$ and $b > 0$. Prove that $\sqrt{a + b} < \sqrt{a} + \sqrt{b}$. [HINT: Use Exercise 1 above.]

2. [**Falkner Section 8 Exercise 10**] Let $a, b \in \mathbb{R}$. Prove that if $0 < a < b$, then $a < \sqrt{ab} < b$.

3. [**Falkner Section 4 Exercise 12**] Show that for each real number x, $\pi + x$ is irrational or $\pi + x$ is rational.