

HOMEWORK 2
MATH 3345 – SPRING 2023 – KUTLER

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 2]** Let P , Q and R be logical sentences. Prove the first Distributive Law

$$P \wedge (Q \vee R) \text{ is logically equivalent to } (P \wedge Q) \vee (P \wedge R)$$

in two ways:

- (a) By means of a truth table;
 - (b) By means of an explanation in words.
2. **[Falkner Section 2 Exercise 6]** Let P and Q be logical sentences. Use a truth table to prove that $\neg(P \Rightarrow Q)$ is logically equivalent to $P \wedge \neg Q$.
3. Write **both** the contrapositive and the converse of each conditional sentence below. Do not worry about the truth value of any of these statements.
- (a) If it is raining, then the ground is wet.
 - (b) If $a = 4$, then $a^2 = 16$.
 - (c) If $a \neq b$, then $a^4 \neq b^4$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Use De Morgan's laws to find a sentence which is logically equivalent to

$$\neg \left[((x > 1) \wedge (x < 3)) \vee ((x \geq 4) \wedge (x < 7)) \right]$$

and which does not use the logical connective " \neg ".

2. Is $(P \Rightarrow Q) \Rightarrow R$ logically equivalent to $P \Rightarrow (Q \Rightarrow R)$? Use a truth table to justify your answer.