

HOMEWORK 20  
MATH 3345 – SPRING 2023 – KUTLER

**Exercises**

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 10 Exercise 19 – modified]** Let  $A$ ,  $B$ , and  $X$  be sets.

- (a) Prove that if  $A \subseteq B$ , then  $X \setminus B \subseteq X \setminus A$ .
- (b) Prove that  $A \subseteq X$  if and only if  $A = X \setminus (X \setminus A)$ . [HINT: Use Homework 19 Exercise 4 to express  $X \setminus (X \setminus A)$  in a simpler form.]
- (c) Suppose  $A \subseteq X$ . Prove that if  $X \setminus B \subseteq X \setminus A$ , then  $A \subseteq B$ .
- (d) Show, by giving an example, that the implication

$$\text{if } X \setminus B \subseteq X \setminus A, \text{ then } A \subseteq B$$

may be **false** if  $A \not\subseteq X$ .

That is, give an example of sets  $A$ ,  $B$ , and  $X$  such that  $X \setminus B \subseteq X \setminus A$  and  $A \not\subseteq B$ .

2. **[Falkner Section 10 Exercise 33(a)–(d)]** Let  $A$ ,  $B$ ,  $C$ , and  $D$  be sets.

- (a) Prove that  $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$ .
- (b) Prove that  $(A \cup B) \times C = (A \times C) \cup (B \times C)$  and  $A \times (B \cup C) = (A \times B) \cup (A \times C)$ .
- (c) Prove that  $(C \times D) \setminus (A \times B) = E \cup F$ , where  $E = (C \setminus A) \times D$  and  $F = C \times (D \setminus B)$ .
- (d) In the special case where  $A = [1, 3] = B$  and  $C = [2, 4] = D$ , draw a picture to illustrate the result you proved for the general case in part (c).

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 10 Exercise 24]** Prove Proposition 10.34(b): Let  $\mathcal{A}$  be a nonempty set of sets and let  $X$  be any object. Then

$$x \notin \left( \bigcap_{A \in \mathcal{A}} A \right) \text{ if and only if there exists } A \in \mathcal{A} \text{ such that } x \notin A.$$

2. **[Falkner Section 10 Exercise 32]** Sketch the rectangle  $[1, 4] \times [2, 3]$  in the coordinate plane. (Shade the set of points that belong to this rectangle.)