Homework 3

Math 3345 – Spring 2023 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 7]** Let x and y be real numbers.
   (a) Let A be the sentence "If $x+y>0$, then $x>0$ or $y>0$." Use Theorem 2.10 and one of De Morgan's laws to show that $\neg A$ is logically equivalent to "$x+y>0$ and $x \leq 0$ and $y \leq 0$." Be careful not to skip any steps.
   (b) Is the sentence A in part (a) true, or is $\neg A$ true? Explain why.
   (c) Let B be the sentence "If $x+y>2$, then $x>2$ or $y>2$." Is B true, or is $\neg B$ true, or is it impossible to say without further information about the specific values of x and y? (Hint: Can you find specific values for x and y for which B is true? If so, give an example of such values. Can you find other specific values for x and y for which $\neg B$ is true? If so, give an example of such values.)
2. **[Falkner Section 2 Exercise 9]** Let $P \operatorname{xor} Q$ mean "P exclusive or Q." In other words, $P \operatorname{xor} Q$ should be true just when **exactly one** of P or Q is true.
   (a) Write out the truth table for $P \operatorname{xor} Q$.
   (b) Show by a truth table that $P \operatorname{xor} Q$ is logically equivalent to $(P \wedge \neg Q) \vee (Q \wedge \neg P)$.
   (c) Show by truth tables that the following four sentences are logically equivalent:
   $$P \operatorname{xor} Q, \qquad \neg(P \Leftrightarrow Q), \qquad (\neg P) \Leftrightarrow Q, \qquad P \Leftrightarrow (\neg Q).$$
   (d) Show by a truth table that $(\neg P) \Leftrightarrow (\neg Q)$ is logically equivalent to $P \Leftrightarrow Q$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 2 Exercise 4]** Suppose that $P \vee Q$ is true and $\neg Q$ is true. Explain why it follows that P must be true.