Homework 6

Math 3345 – Spring 2023 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. Let $A(x, y)$ be the conditional sentence

   $$\text{If } x = 3 \text{ and } y = 5, \text{ then } xy = 15.$$

   Although most people probably read this sentence as a straightforwardly true statement, it is technically a function of the variables x and y (it's just that "$x = 3$ and $y = 5$" is false for most choices of x and y). To produce an unambiguously true or false sentence, we should bound these variables with quantifiers.

   (a) Use one of the generalized De Morgan's laws to write the negation of the sentence $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})\, A(x, y)$.

   (b) Is $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})\, A(x, y)$ true or false? Explain your answer.

   (c) Let $B(x, y)$ be the converse of $A(x, y)$. Is $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})\, B(x, y)$ true or false? Explain your answer.

2. **[Falkner Section 3 Exercise 7]** Let P be the sentence

   $$(\exists x \in \mathbb{R})\big(x \geq 0 \text{ and } \sqrt{x+2} < \sqrt{x} + \sqrt{2}\big).$$

   (a) Use one of the generalized De Morgan's laws and one of the ordinary De Morgan's laws to show that $\neg P$ is logically equivalent to

   $$(\forall x \in \mathbb{R})\big(x < 0 \text{ or } \sqrt{x+2} \geq \sqrt{x} + \sqrt{2}\big).$$

   (b) Is P true or false? Provide a proof for your answer.

3. **[Falkner Section 3 Exercise 10]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your statement.

   (a) $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = x)$.

   (b) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = x)$.

   (c) $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = 0)$.

   (d) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = 0)$.

   (e) $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(xy = 1)$.

   (f) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy = 1)$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 3 Exercise 8]** Which of the variables x and y is free in the sentence $P(x, y)$. Answer the same question about each of the four sentences $(\exists y)P(x, y)$, $(\forall x)(\exists y)P(x, y)$, $(\forall x)P(x, y)$, and $(\exists y)(\forall x)P(x, y)$.

2. **[Falkner Section 3 Exercise 9]** See book for problem statement.