

HOMEWORK 7  
MATH 3345 – SPRING 2023 – KUTLER

**Exercises**

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 3 Exercise 14]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your answer.

- (a)  $(\exists! x \in \mathbb{R})(2x + 7 = 3)$ .
- (b)  $(\exists! x \in \mathbb{R})(x^2 - 4x + 3 < 0)$ .
- (c)  $(\exists! x \in \mathbb{Z})(x^2 - 4x + 3 < 0)$ .
- (d)  $(\exists! x \in \mathbb{R})(x^2 - 4x + 4 = 0)$ .
- (e)  $(\exists! x \in \mathbb{R})(x^2 - 4x + 5 = 0)$ .
- (f)  $(\forall x \in \mathbb{R})(\exists! y \in \mathbb{R})(x + y = 0)$ .
- (g)  $(\forall x \in \mathbb{R})(\exists! y \in \mathbb{R})(xy = 1)$ .
- (h)  $(\forall x \in \mathbb{R})[\text{if } x \neq 0, \text{ then } (\exists! y \in \mathbb{R})(xy = 1)]$ .
- (i)  $(\forall x \in \mathbb{R})(\exists! y \in \mathbb{R})(xy = 0)$ .
- (j)  $(\forall x \in \mathbb{R})[\text{if } x \neq 0, \text{ then } (\exists! y \in \mathbb{R})(xy = 0)]$ .

2. **[Falkner Section 5 Exercise 2]** Prove by induction that for each  $n \in \mathbb{N}$ ,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

## Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Read the (incorrect) proof given in [**Falkner Section 5 Exercise 8**], which purports to prove by induction that all horses have the same color.
  - (a) Modify this proof to obtain (incorrect) proofs of the following (false) statements:
    - i. All students at OSU have the same favorite food.
    - ii. All songs are in the same key.
    - iii. All rivers flow in the same direction.
    - iv. All mountains have the same height.
  - (b) Each of these proofs makes the same error. What is it?