

HOMEWORK 9
MATH 3345 – SPRING 2023 – KUTLER

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 4 Exercise 5]** Let x and y be integers. Prove the following statements.
 - (a) If xy is even, then x is even or y is even.
 - (b) If xy is odd, then x is odd and y is odd.
2. **[Falkner Section 4 Exercise 6]** Let a be an integer. Use the results of the previous exercise to prove the following statements.
 - (a) If a^2 is even, then a is even.
 - (b) If a^2 is odd, then a is odd.

3. **[Integers Exercise 1]** Prove Lemma 4: For any $a \in \mathbb{Z}$, $-(-a) = a$.

You may **only** assume the axioms for the integers and Lemmas 1–3 on the Integers handout. You may find the Additive Inverses axiom and Lemma 2 (Uniqueness of Additive Inverses) particularly useful.

4. **[Integers Exercise 2]** Prove Lemma 7: For any $a, b \in \mathbb{Z}$, if $a \cdot b = 0$, then $a = 0$ or $b = 0$.

You may use only the axioms for the integers and Lemmas 1 through 6. [HINT: Prove the contrapositive statement. If an integer x is not zero, then by the Trichotomy axiom, either $x \in \mathbb{N}$ or $-x \in \mathbb{N}$. Now, consider cases and use the Positive Closure axiom.]

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Integers Exercise 3]** Prove Lemma 9: For any $a, b \in \mathbb{Z}$, exactly one of the following is true: (i) $a < b$, or (ii) $a = b$, or (iii) $b < a$.

[HINT: You will need to use the Trichotomy axiom, the Distributive Law, and Lemmas 4 and 5.]