Logical Sentences

A logical sentence is a sentence with a well-defined <u>truth value</u> (True or False).

<u>Ex</u>: "OSU is in Columbus, OH." (T)

"$2+2=5$." (F)

"$x>7$" (Depends on variable x)

<u>Non-Ex</u>: "Go outside."
"Are you cold?"

It is clear from these examples that logical sentences require <u>precise definitions</u>.

Propositional Calculus

← Nothing to do with differential/integral calculus

How to "build" new sentences from existing ones?

Use **logical connectives**.

Logical Connective	Symbol	Plain English
negation	$\neg$	"not"
conjunction	$\wedge$	"and"
disjunction	$\vee$	"or" (inclusive)
implication	$\Rightarrow$	"if-then"
biconditional	$\Leftrightarrow$	"if and only if"

Let P, Q, R, ... stand for sentences.

Ex: P = "It is Friday."

Q = "We're having fun in Math 3345."

$\neg P$, $P \wedge Q$, $P \Rightarrow Q$, etc.

1. <u>Negation</u>: $\neg$ means "not"

The negation $\neg P$ has the opposite truth value as P.
So if P is true, then $\neg P$ is false,
if P is false, then $\neg P$ is true.

Summarize this in a truth table:

P	$\neg P$
T	F
F	T

<u>Ex</u>: What is $\neg(\neg P)$? Make another truth table:

P	$\neg P$	$\neg(\neg P)$
T	F	T
F	T	F

So P and $\neg(\neg P)$ always have the same truth value. We say they are <u>logically equivalent</u> and write

$$P \equiv \neg(\neg P).$$

"is logically equivalent to"

② Conjunction: $\wedge$ means "and"

$P \wedge Q$ is true exactly when **both** P and Q are true.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex: 2 is even and 3 is odd. T
2 is even and 3 is even. F
2 is odd and 3 is odd. F