Warm-Up: Use induction to show

$$1 + 3 + 3^2 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}$$

for every non-negative integer n.

Parity

Def: Let $n \in \mathbb{Z}$ be an integer. Then

① n is **even** if $n = 2k$ for some $k \in \mathbb{Z}$.

② n is **odd** if $n = 2\ell + 1$ for some $\ell \in \mathbb{Z}$

These are $\exists$ statements!

Thm: For any $x, y \in \mathbb{Z}$,

① If x is odd and y is odd, then $x+y$ is even.

② If x is even and y is odd, then $x+y$ is odd.

③ If x is even and y is even, then $x+y$ is even.

Proof: ① Suppose x and y are both odd. Then there exist integers k and l such that

$$x = 2k+1$$

and

$$y = 2l+1$$

Thus,

$$\begin{aligned} x+y &= (2k+1) + (2l+1) \\ &= 2(k+l+1). \end{aligned}$$

Since $k+l+1 \in \mathbb{Z}$, this proves $x+y$ is even.

② and ③ left as exercises. ∎

Is there any integer which is <u>both</u> even and odd?

This would imply 1 is even!

(Or, equivalently, $\frac{1}{2} \in \mathbb{Z}$.)

How do we know this isn't so?