HW:
- neat/legible
- name at top!
- staple or paperclip if necessary
- complete sentences
- audience is your classmates
- collaborate responsibly

Last time: $\neg$ and $\wedge$

Warm-Up: Check that

- $P \wedge Q \equiv Q \wedge P$
- $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

That is, $\wedge$ is commutative and associative.

③ Disjunction: $\vee$ means "or" (inclusive)

$P \wedge Q$ is true exactly when at least one of P or Q is true.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Ex: 2 is even or 3 is odd. T
2 is even or 3 is even. T
2 is odd or 3 is even. F

Note: $P \vee Q \equiv Q \vee P$

$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$$

How do the operations $\neg$, $\wedge$, $\vee$ interact with one another?

Ex: Let

$$P = \text{"}m \text{ is even."}$$
$$Q = \text{"}n \text{ is odd."}$$

Then

$P \wedge Q$ is "m is even and n is odd."

$\underbrace{\text{This becomes false}}_{\text{i.e. } \neg(P \wedge Q) \text{ is true}}$ if $\underbrace{m \text{ is not even } \underline{\text{OR}} \ n \text{ is not odd.}}_{\equiv \neg P \vee \neg Q}$

In general, we have:

Thm (DeMorgan's Laws) Let P and Q be sentences. Then

(a) $\neg(P \wedge Q)$ is logically equivalent to $\neg P \vee \neg Q$

(b) $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$

Proof of (a):

By truth table:

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

So we see $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$.

We can also prove this by giving an explanation in words:

We wish to show $\neg(P \wedge Q)$ always has the same truth value as $\neg P \vee \neg Q$.

First, suppose $\neg(P \wedge Q)$ is true. Then $P \wedge Q$ is false, so at least one of P or Q is false.

But this means at least one of $\neg P$ or $\neg Q$ is true, so $\neg P \vee \neg Q$ is true.
In words:

Next, suppose $\neg(P \wedge Q)$ is false. Tlen $P \wedge Q$ is true, so both P and Q are true.

Now, both $\neg P$ and $\neg Q$ will be false, meaning $\neg P \vee \neg Q$ is false as well.

(b) HW 1