Warm-Up: Let P, Q be sentences.

Find a sentence using only the logical connectives $\neg$ and $\wedge$ which is logically equivalent to $P \vee Q$.

De Morgan tells us how $\neg$ interacts with $\wedge$ and $\vee$.

How do $\wedge$ and $\vee$ interact with each other?

Thm (Distributive Laws)
Let P, Q, R be sentences. Then

(a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

(b) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Ex: P = "It is a nice day"
Q = "I will go for a walk"
R = "I will eat outside"

<u>Proof of (b)</u>: We want to show these two sentences always have the same truth value.

First, suppose $P \vee (Q \wedge R)$ is true. Then either

- P is true

or

- $Q \wedge R$ is true, which means <u>both</u> Q and R are true

(or both).

In either case, $P \vee Q$ is true <u>and</u> $P \vee R$ is true, so $(P \vee Q) \wedge (P \vee R)$ is true.

The other possibility is that $P \vee (Q \wedge R)$ is false. This means that <u>both</u>

- P is false

and

- $Q \wedge R$ is false, which in turn means at least one of Q or R is false.

But then at least one of $P \vee Q$ or $P \vee R$ is false. So $(P \vee Q) \wedge (P \vee R)$ is false. $\blacksquare$

As a truth table:

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Another logical connective:

(4) **Implication**: $\Rightarrow$ means "implies" or "if-then"

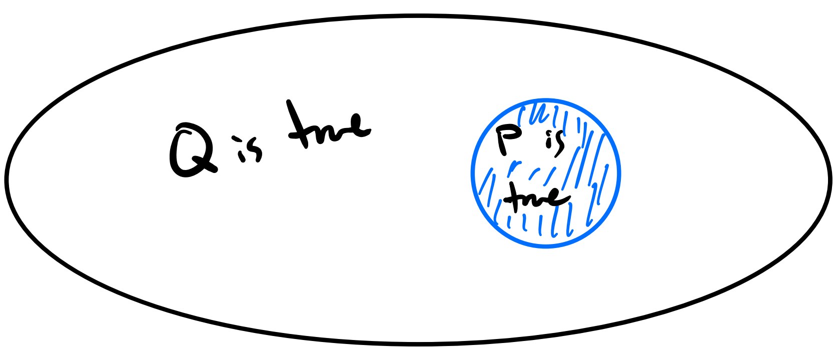
$P \Rightarrow Q$ means "if P is true, then Q is true"

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Why do the last 2 rows make sense?

Another perspective:

$P \Rightarrow Q$ is true when Q is "at least as true" as P.



Ex: If it's raining, then the ground is wet. T

If $x = 3$, then $x^2 = 9$. T

If $x^2 = 9$, then $x = 3$. F

If $0 > 1$, then $3^2 = 9$. T

If $0 > 1$, then the sun will explode today at 5 pm. T

Note:
- If P is false, then $P \Rightarrow Q$ is true.
- If Q is true, then $P \Rightarrow Q$ is true.

In fact,

Prop: Let P and Q be sentences. Then

$$P \Rightarrow Q \equiv \neg P \vee Q$$

Proof: The only situation in which $P \Rightarrow Q$ is false is if P is true <u>and</u> Q is false.

This is precisely when $\neg P \vee Q$ is false.

In all other cases, both $P \Rightarrow Q$ and $\neg P \vee Q$ are true.

$\blacksquare$

Alternatively,

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Corollary: $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$

Proof: $\neg(P \Rightarrow Q) \equiv \neg(\neg P \vee Q)$

$\equiv \neg(\neg P) \wedge \neg Q$ (DeMorgan)

$\equiv P \wedge \neg Q$ (Double negation)

Ex: The negation of

"If it is Wednesday, then I will attend class"

is logically equivalent to

"It is Wednesday and I will not attend class."