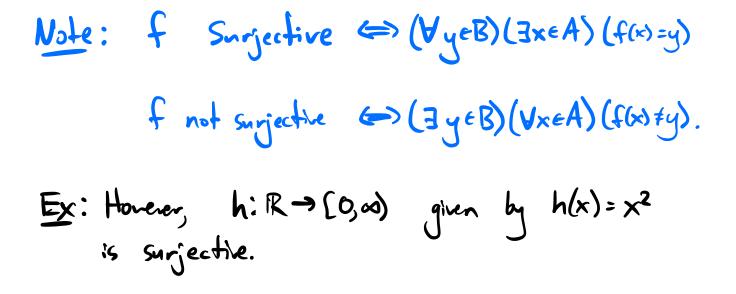
Def: Let $f:A \rightarrow B$ be a function. We say f is a <u>surjection</u> if for all $y \in B$, there exists $x \in A$ such that f(x) = y. Also say: f is <u>surjective</u>, f is <u>onto</u>. Equivalently: f: A > B is surjective (=) Rug(f) = B Ex: Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3$. Then f is a surjection. Proof: Let yER. Set x= 3/y ER. Then $f(3y) = (3y)^{3} = y.$ Ex: Let $g: \mathbb{R} \to \mathbb{R}$ be given by $g(x) = x^2$. Then g is not surjective.

Proof: Consider - I ER. Then for all $x \in \mathbb{R}$, $g(x) = x^2 \neq -1$.



Injections

<u>Def</u>: A function $f: A \rightarrow B$ is an <u>injection</u> if for all $x_{i,x_2} \in A$, if $f(x_i) = f(x_2)$, then $x_i = x_2$.

Also say: fis injective, fis <u>one-to-one</u>.

Ex: f: $\mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3$ is an injection. Proof: Let $x_{1,x_2} \in \mathbb{R}$ and suppose $x_1^3 = x_2^3$. Then $3\sqrt{x_1^3} = 3\sqrt{x_2^3}$, i.e. $x_1 = x_2$.

Ex: g:
$$\mathbb{R} \rightarrow \mathbb{R}$$
 given by $g(x) = x^2$ is
not an injection.
Proof: Considur $-1, 1 \in \mathbb{R}$. Then $-1 \neq 1$, but
 $g(-1) = (-1)^2 = 1 = (1)^2 = g(1)$.

$$\underbrace{N_{o}}_{T} \text{ injective } \iff (\forall x_{1}, x_{2} \in A) \begin{bmatrix} f(x_{1}) = f(x_{2}) \\ T \end{bmatrix} \approx x_{1} = x_{2} \end{bmatrix}$$
$$\iff (\forall x_{1}, x_{2} \in A) \begin{bmatrix} x_{1} \neq x_{2} \implies f(x_{1}) \neq f(x_{2}) \end{bmatrix}$$

f not injective
$$\iff (\exists x_1, x_2 \in A) [x_1 \neq x_2 \text{ and } f(x_1) = f(x_2)]$$

Bijections
Def: A function f: A → B is a bijection if
it is both a surjection and an injection.
f surjective
$$\iff (\forall y \in B) (\exists x \in A) [f(x) = y]$$

f injective $\iff (\forall x_{1,x_{2}} \in A) [f(x_{1}) = f(x_{2}) \Rightarrow x_{1} = x_{2}]$
 $\iff (\forall y \in B) (\forall x_{1,x_{2}} \in A) [f(x_{1}) = f(x_{2}) \Rightarrow x_{1} = x_{2}]$
 $\iff (\forall y \in B) (\forall x_{1,x_{2}} \in A) [f(x_{1}) = y \land f(x_{2}) = y]$
Together, we get the following:
Lemma: Let f: A → B be a function. Then
f is a bijection if and only if for
every $y \in B$, there exists a unique
 $x \in A$ such that $f(x) = y$.
Ex: (: Z → Z = Ex or | x \in Z = Here is a

 $\underbrace{\mathsf{E} \mathsf{X}}_{\mathsf{n}} : \widehat{\mathsf{F}} : \mathbb{Z} \to \mathbb{Z}_{\mathsf{n}} \text{ For each } \mathsf{y} \in \mathbb{Z}_{\mathsf{n}}, \text{ there is a} \\ \mathsf{n} \longmapsto \mathsf{n}^{\mathsf{H}}_{\mathsf{n}} \text{ unique } \mathsf{x} \in \mathbb{Z}_{\mathsf{n}} \text{ such } \text{ that } f(\mathsf{x}) = \mathsf{y}, \\ \mathsf{namely} \quad \mathsf{x} = \mathsf{y} - \mathsf{I}_{\mathsf{n}}.$