

Warm-Up: Let x and y be real numbers.

Let S be the conditional sentence

"If $xy > 0$, then $x > 0$ and $y > 0$."

- Is S true or false?
 - Write the converse and contrapositive of S . Are these sentences true or false?
 - Write the negation $\neg S$ as an "and" sentence.
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Another perspective on conditional proof:

To prove $A \Rightarrow C$ is true, it is enough to show that it is impossible for A to be true and C to be simultaneously false.

To do this, either

- Assume A is true and show C must also be true.

OR

- Assume C is false and show A must also be false. [i.e. prove $\neg C \Rightarrow \neg A$]

Tautologies

A sentence is called a tautology if it is always true for structural reasons (i.e. because of how it is constructed using \neg , \wedge , \vee , \Rightarrow , and \Leftrightarrow).

Ex: "Modus ponens"

$$(P \Rightarrow Q) \wedge P \Rightarrow Q$$

is a tautology.

Proof: Assume $(P \Rightarrow Q) \wedge P$ is true.

[Again, if it's false there's nothing to do.]

Then both of $P \Rightarrow Q$ and P are true, so Q must be true. ▀

Modus ponens is a common step in logical reasoning.

Ex: If it is raining, then the ground is wet. It is raining.

Therefore, the ground is wet.

Ex: $P \vee \neg P$ is a tautology.

"The law of the excluded middle."

Ex: Show that the sentence

$$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$$

is a tautology.

"Hypothetical syllogism"

Proof: Suppose $(P \Rightarrow Q) \wedge (Q \Rightarrow R)$ is true. Thus, $P \Rightarrow Q$ and $Q \Rightarrow R$ are both true.

We want to argue that $P \Rightarrow R$ must be true.

Assume P is true. [If not, $P \Rightarrow Q$ is true vacuously and we are done.]

Since $P \Rightarrow Q$ and P are both true, Q must be true.

Since $Q \Rightarrow R$ and Q are both true, R must be true.

Therefore, $P \Rightarrow R$ is true, as desired. ■