1 (Judson 1.6 & 1.16) Let A and B be sets.
Prove the following equalities by showing a set containment in both directions.

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

2 (Judson 1.20) Provide examples of functions satisfying the given conditions.

(a) $f\colon \mathbb{N} \to \mathbb{N}$ is injective but not surjective.

(b) $g\colon \mathbb{N} \to \mathbb{N}$ is surjective but not injective.

3 Let X and Y be sets with $|X| = 4$ and $|Y| = 7$.

(a) How many subsets does X have? How many of these are *proper* subsets?

(b) What is $|X \times Y|$?

(c) How many functions are there from X to Y?

(d) How many functions are there from Y to X?

4 Let $f: X \to Y$ and $g: Y \to Z$ be maps of sets. The composition $g \circ f: X \to Z$ may be diagrammed as

$$X \xrightarrow{f} Y \xrightarrow{g} Z.$$

(a) Prove that if f and g are each injective, then $g \circ f$ is injective.

(b) Prove that if f and g are each surjective, then $g \circ f$ is surjective.

(c) Conclude that if f and g are each bijections, then $g \circ f$ is a bijection.