

1 Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 4 & 1 \end{pmatrix}.$$

Write the following permutations in cycle notation, as a product of disjoint cycles.

(a)  $\sigma$

(b)  $\tau$

(c)  $\sigma\tau$

(d)  $\tau\sigma$

(e)  $\sigma^2$

2 Write each of the following permutations as a product of disjoint cycles in  $S_6$ .

(a)  $(1\ 2)(2\ 3)(3\ 4)(4\ 5)(5\ 6)$

(b)  $(3\ 4\ 5)^{-1}$

(c)  $(3\ 4\ 5)(2\ 5\ 6\ 4)(3\ 5\ 4)$

3 Let  $\sigma = (a_1 a_2 \cdots a_k)$  be a cycle of length  $k$ .

Prove that  $\sigma$  can be written as a product of  $k - 1$  transpositions.

[HINT: Look at problem 2(a) for inspiration.]