

1 (Judson 5.3) Express the following permutations as products of transpositions and identify them as even or odd.

(a) $(1\ 4\ 3\ 5\ 6)$

(b) $(1\ 5\ 6)(2\ 3\ 4)$

(c) $(1\ 4\ 2\ 6)(1\ 4\ 2)$

(d) $(1\ 7\ 2\ 5\ 4)(1\ 4\ 2\ 3)(1\ 5\ 4\ 6\ 3\ 2)$

(e) $(1\ 4\ 2\ 6\ 3\ 7)$

2 Recall that

$$D_4 = \langle r, s \mid r^4 = s^2 = 1, rs = sr^{-1} \rangle.$$

- (a) Find all cyclic subgroups of D_4 .
- (b) Find all proper subgroups of D_4 which are not cyclic.
- (c) Draw the subgroup lattice of D_4 .

3 (Judson 5.29 – modified) Let G be a group. The **center** of G is defined to be

$$Z(G) = \{z \in G \mid zg = gz \text{ for all } g \in G\}.$$

- (a) Prove that $Z(G)$ is a subgroup of G .
- (b) Prove that $Z(G)$ is abelian.
- (c) Let $n \geq 3$ be an integer. Prove that

$$Z(D_n) = \begin{cases} \{1\} & \text{if } n \text{ is odd;} \\ \{1, r^{n/2}\} & \text{if } n \text{ is even.} \end{cases}$$