1 Let $Q_8 = \{1, -1, i, -i, j, -j, k, -k\}$, which is a group under multiplication where

- 1 is the identity element.
- $(-1) \cdot i = -i$, and similarly for $-j$ and $-k$.
- $i^2 = j^2 = k^2 = -1$.
- $ij = k$, $jk = i$, and $ki = j$.
- $ji = -k$, $kj = -i$, and $ik = -j$.

(a) Use the above information to write the Cayley table (multiplication table) for Q_8.

(b) Find all cyclic subgroups of Q_8.

(c) Prove that every proper subgroup of Q_8 is cyclic.

(d) Draw the subgroup lattice of Q_8.

2 Let G be a group and $x \in G$. Define the **conjugacy class** containing x to be the set

$$\{gxg^{-1} \mid g \in G\}.$$

[NOTE: On Exam 1, you proved that the relation

$$x \sim y \text{ if and only if there exists } g \in G \text{ such that } y = gxg^{-1}$$

is an equivalence relation on G. The conjugacy class containing x is just the equivalence class $[x]$ under this relation.]

Compute all of the conjugacy classes in the following groups.

(a) $\mathbb{Z}_8$

(b) D_4

(c) Q_8