1 Let G be a group, and suppose $H \leq G$ is a subgroup of index 2. Prove that $gH = Hg$ for every $g \in G$.

2 Let $n \in \mathbb{N}$.

(a) Show that if $n \geq 3$, then $U(n)$ contains an element of order 2.

(b) Recall that $|U(n)| = \phi(n)$ is equal to the number of integers a such that $1 \leq a \leq n$ and $\gcd(a, n) = 1$.

Use part (a) and Lagrange's theorem to prove that $\phi(n)$ is even if $n \geq 3$.

3 Let G be a group with $|G| = 15$

(a) What are the possible orders of elements in G?

(b) Prove that G must have an element of order 3.