**1** Identify every normal subgroup of $D_4$.
**Note:** You already found all subgroups of $D_4$ on HW 11. The subgroup lattice also appears in the lecture notes for Day 25.

**2** Let $G$ be a group, and let $N \trianglelefteq G$ and $H \leq G$. Show that

$$HN = \{hn \mid h \in H, n \in N\}$$

is a subgroup of $G$.

**3** Prove that if $N \trianglelefteq G$ and $H$ is any subgroup of $G$, then $N \cap H \trianglelefteq H$.