

**1** Let  $G$  be a finite group. Suppose  $H \leq G$  is a subgroup, and that it is the only subgroup of order  $|H|$ . Prove that  $H$  is normal.

HINT: On HW 6 you proved that for any  $g \in G$ ,

$$gHg^{-1} = \{ghg^{-1} \mid h \in H\}$$

is also a subgroup of  $G$ . What is  $|gHg^{-1}|$ ?

**2**

(a) Prove that  $\langle -1 \rangle$  is a normal subgroup of  $Q_8$ .

(b) Prove that  $Q_8 / \langle -1 \rangle \cong V_4$ .