1 Let G be a group and let $N \trianglelefteq G$ be a normal subgroup.
Prove that the cosets xN and yN commute in G/N if and only if $x^{-1}y^{-1}xy \in N$. (The element $x^{-1}y^{-1}xy$ is called the **commutator** of x and y.)

2 Let $H = \{\text{id}, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\} \leq S_4$. (You should convince yourself that H is in fact a subgroup of S_4; however, you do not need to prove it here.)

(a) Prove that $H \trianglelefteq S_4$.

(b) What is the cardinality of S_4/H? Write a complete list (with no repetitions) of the cosets of H in S_4.

(c) Write the group operation table for the quotient group S_4/H.

(d) Give an example of a homomorphism with domain S_4 and kernel H.

3 Consider the function $\varphi\colon \mathbb{R}^\times \to \mathbb{R}^\times$ defined by $\varphi(x) = |x|$.

(a) Prove that φ is a homomorphism.

(b) Find the kernel and the image of φ.

(c) What does the First Isomorphism Theorem say when applied to φ?

(d) Describe the fibers of φ.

4 Let G be a group. Recall from HW 15 Problem 3 the inner automorphism group

$$\mathrm{Inn}(G) = \{\gamma_g \mid g \in G\},$$

where γ_g is the automorphism

$$\begin{aligned} \gamma_g \colon G &\to G \\ x &\mapsto gxg^{-1} \end{aligned}$$

(a) Prove that

$$\begin{aligned} \Gamma \colon G &\to \mathrm{Inn}(G) \\ g &\mapsto \gamma_g \end{aligned}$$

is a surjective homomorphism.

(b) Use the First Isomorphism Theorem to prove that $G/Z(G) \cong \mathrm{Inn}(G)$.