1 (Judson 1.21) Prove the relation defined on $\mathbb{R}^2$ by

$$(x_1, y_1) \sim (x_2, y_2) \quad \text{if and only if} \quad x_1^2 + y_1^2 = x_2^2 + y_2^2$$

is an equivalence relation.

2 Let $f\colon X \to Y$ be a surjective map of sets. For each $y \in Y$, the **fiber of** f **over** y is the set

$$f^{-1}(y) = \{x \in X \mid f(x) = y\}.$$

[CAUTION: The fiber $f^{-1}(y)$ is a *subset* of X.]

(a) For example, consider the surjection $f\colon \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ given by $f(x,y) = x^2 + y^2$. For each $r \in \mathbb{R}_{\geq 0}$, describe the fiber $f^{-1}(r)$.

(b) Let $f\colon X \to Y$ be a surjective map of sets. Prove that the relation

$$x_1 \sim x_2 \quad \text{if and only if} \quad f(x_1) = f(x_2)$$

is an equivalence relation on the set X. Prove that the equivalence classes are the fibers of f.

3 (Judson 2.11) Let $x \geq 0$ be a real number. Use induction to prove that

$$(1+x)^n \geq 1 + nx$$

for every integer $n \geq 0$.