1 Let A and B be groups, and consider the direct product $G = A \times B$.

(a) Prove that
$$N = \{(e_A, b) \in A \times B \mid b \in B\}$$
is a subgroup of G, where e_A is the identity element of A.

(b) Prove that $N \cong B$.

(c) Prove that $N \trianglelefteq G$.

(d) Prove that $G/N \cong A$.

2 Let G be a finite group, and let $N \trianglelefteq G$ be a normal subgroup. Let $g \in G$.

(a) Explain why the element gN in the group G/N has finite order.

(b) Prove that the order of gN in G/N divides the order of g in G.