

1 Define binary operations $*$, \diamond , and \star on the set \mathbb{R} of real numbers as follows:

(a) $a * b = \frac{ab}{3}$.

(b) $a \diamond b = \min(a, b)$.

(c) $a \star b = a + b - ab$.

For each of these three binary operations, answer the following questions.

- i. Is the binary operation associative?
- ii. Is the binary operation commutative?
- iii. Is there an identity element for the operation?
- iv. If there is an identity element, which elements of \mathbb{R} have an inverse?

2 Let $(G, *)$ be a group. Prove that if $g, h \in G$ satisfy $g * h = e$, then $h * g = e$ also.