1 Let G be a group, and let $g \in G$. Prove that $|g| = |g^{-1}|$.

2

(a) Find the order of every element in $\mathbb{Z}_{18}$. For which $a \in \mathbb{Z}_{18}$ is $\langle a \rangle = \mathbb{Z}_{18}$?

(b) Find the order of every element in $U(18)$. Does there exist an element $a \in U(18)$ such that $\langle a \rangle = U(18)$?

3 Let G be a group, and suppose that G has no proper non-trivial subgroups. That is, the only subgroups of G are G itself and $\{e\}$, where $e \in G$ is the identity element. Prove that G is cyclic and $|G| = p$ for some prime number p.