1 Let G be a group, and let $H \leq G$ and $K \leq G$ be subgroups.

(a) Prove that $H \cap K \leq G$.

(b) Prove that $H \cup K \leq G$ if and only if either $H \subseteq K$ or $K \subseteq H$.

2 Let S be a set, and let G be the set of all bijections $f\colon S \to S$. It follows from Math 3345 that G is a group under function composition $\circ$. Indeed

- **$\circ$ is a binary operation on G:** If $f\colon S \to S$ and $g\colon S \to S$ are bijections, then $f \circ g\colon S \to S$ is a bijection.
- **$\circ$ is associative:** $(f \circ g) \circ h = f \circ (g \circ h)$.
- **Identity:** The identity function $\mathrm{id}_S\colon S \to S$, defined by $\mathrm{id}_S(x) = x$ for all $x \in S$, is a bijection and it is the identity element for $\circ$.
- **Inverses:** For any bijection $f\colon S \to S$, the inverse function $f^{-1}\colon S \to S$ is also a bijection, and it is the inverse of f with respect to $\circ$.

You don't need to prove any of these statements, but they should all sound familiar!

In the questions below, we consider the case where $S = \{1, 2, 3\}$.

(a) Compute $|G|$.

(b) Find $f, g \in G$ such that $f \circ g \neq g \circ f$. Thus, G is a non-abelian group.

(c) Show that if $f \in G$ is not the identity, then either $f^2 = \mathrm{id}_S$ or $f^3 = \mathrm{id}_S$.