

Math 4580 - Abstract Algebra I

Syllabus

- Website - must check
- Homework - due Tues (my office 3 pm)
 - + Fri (beginning of class)
 - HW 1 due Tues, 8/29
 - Reading is part of homework
 - Guidelines
 - LaTeX optional
 - Rewrites (details to come)
 - Collaboration encouraged!
- 3 exams
- Grades - 20% HW, 20% each exam,
20% $\max\{\text{HW}, \text{best exam}\}$
- Attendance - crucial

What is algebra?

My answer: The study of structure in mathematical objects

Ex: The equation $x^2 + 1 = 0$ has

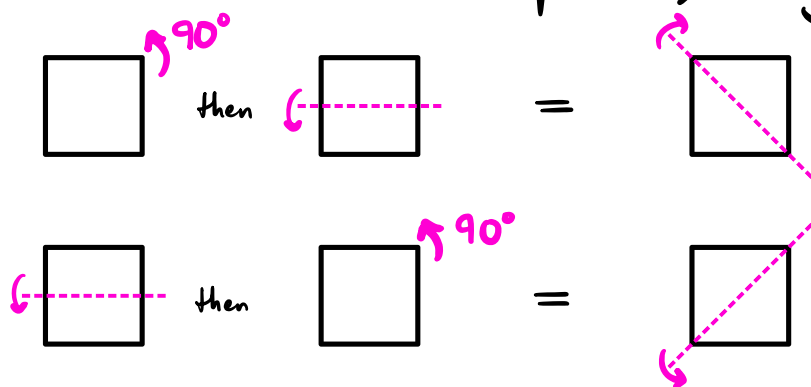
- no solution in \mathbb{R} ,
- two solutions in \mathbb{C} ($x = \pm i$).

This tells us something about the "structure" of \mathbb{R} and \mathbb{C} .

Ex: Symmetries of a square

- 4 rotations (incl. by 0°)
- 4 reflections.

These can be composed, e.g.,



Ex: Rubik's cube has

43,252,003,274,489,856,000

↑ 43 quintillion

Symmetries.

Also not commutative, e.g., $RU \neq UR$.

Review of set theory

- Precise definition requires axioms.
- Informally, a set is a collection of objects, called elements of the set.

Notation: $a \in S$

- Sets are well-defined: for any set S and object a , either $a \in S$ or $a \notin S$.
- There is precisely one set with no elements, the empty set \emptyset .

Notation: $S = \{ \dots \}$
↑ list of all elements

• set-builder notation

$$S = \{ x \mid P(x) \}$$

↑ a true/false statement

Ex: $E = \{ 2, 4, 6, 8, \dots \} = \{ x \in \mathbb{N} \mid x \text{ is even} \}$

Subsets

If A and B are sets, then

- A is a subset of B if

$$x \in A \Rightarrow x \in B$$

for every object x . Equivalently, every element of A is also an element of B .

Notation: $A \subseteq B$ or $A \subset B$

- A and B are equal ($A = B$) if $A \subseteq B$ and $B \subseteq A$.

Equivalently, $A = B$ means that

$$x \in A \Leftrightarrow x \in B$$

for every object x .

- A is a proper subset of B if $A \subseteq B$ and $A \neq B$.

Notation: $A \subsetneq B$