$$D_n = \{e, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}$$

where r = $2\pi/n$ radian rotation counterclockwise

s = reflection across line of symmetry through vertex v.

Warm-Up: Draw the 10 elements of D_5 as symmetries of ⬠.

Prop: In D_n,

① $|s| = |sr^i| = 2$

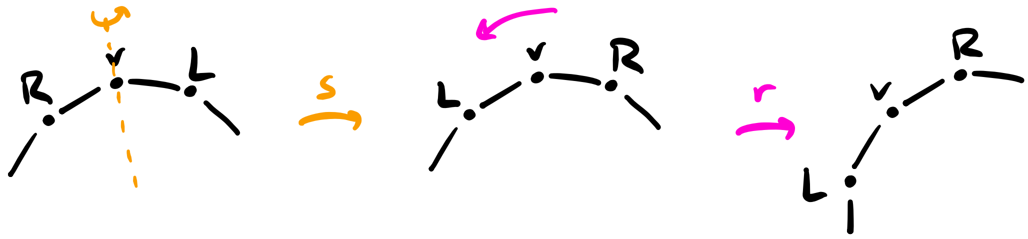
② $rs = sr^{-1}$ (equivalently, $srs = r^{-1}$)

③ $r^i s = sr^{-i}$.

Proof: ① $s \neq e$ but $s^2 = e$ (look at v and its neighbors), so $|s| = 2$.

We'll come back for sr^i.

② rs:



sr^{-1}:

③ Induct on i:

$$\begin{aligned} r^{i+1}s &= r(r^i s) = r(s r^{-i}) \\ &= (rs)r^{-i} \\ &= (sr^{-1})r^{-i} \\ &= sr^{-i}. \end{aligned}$$

① Continued: $sr^i \neq e$, but

$$(sr^i)^2 = sr^isr^i = ssr^{-i}r^i = s^2 = e,$$

so $|sr^i| = 2$. ∎

We have the <u>presentation</u>

$$D_n = \langle \underbrace{r, s}_{\text{Generators}} \mid \underbrace{r^n = s^2 = e, \quad rs = sr^{-1}}_{\text{Relations}} \rangle.$$

<u>Ex</u>: In D_5,

$$
\begin{aligned}
\underline{s^3} r s r^3 \underline{s^2} r^4 s &= srs(r^3 r^4)s \\
&= srs\underline{r^7}s \\
&= srs(r^2 s) \\
&= sr\underline{ss}\,\underline{r^{-2}} \\
&= srr^3 \\
&= sr^4.
\end{aligned}
$$