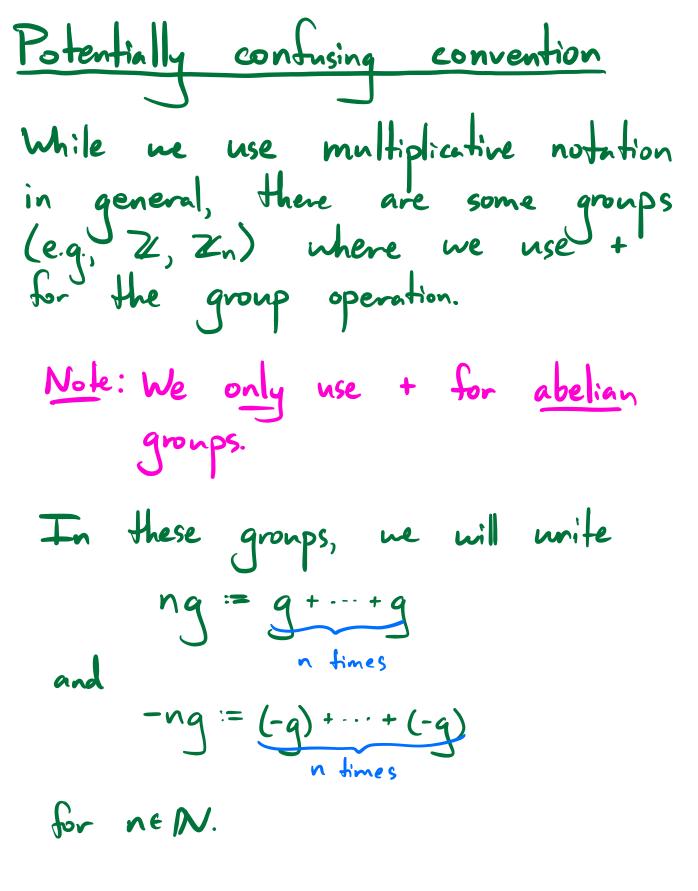
Order

Recall: L geG.	et G be a group	and
For	n EIN, ne vill unite	
and	$g^{n} := g \cdot g \cdot g \cdot g$ n times $\overline{g}^{n} := \overline{g}^{1} \cdot \overline{g}^{1} \cdot \cdots \overline{g}^{1}$.	

We will also write $g^{\circ} := e$.

abelian, (gh)" ≠ gⁿhⁿ in general.



Also, Og = e.

Def: Let G be a group and geG. The order of g is the smallest positive integer n such that $g^n = e$. We write |g| = n. If no such possitive integer exists, ne say g has infinite order and write $\|g\| = \infty$.

Def: Let G be a group. If |G| = n for some $n \in N$, then ne say G is a finite group and that G has order n. If IGI is infinite, ne say G is an infinite group. We also say that it is a group of infinite order.

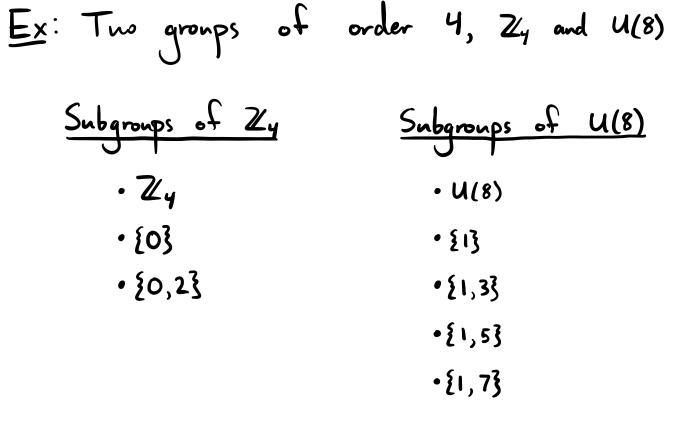
 $E_{X}: |Z_{4}| = 4$, and |0|=1, |1|=4, |2|=2, |3|=3. $E_{x}: |U(8)| = 4$, and |1| = 1, |3| = 2, |5| = 2, |7| = 2. Ex: Z is infinite. 101=1, and 1n1=00 if n70. Ex: GL, (R) is infinite. $\left| \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right| = 4, \qquad \left| \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \right| = \infty.$ Check these!

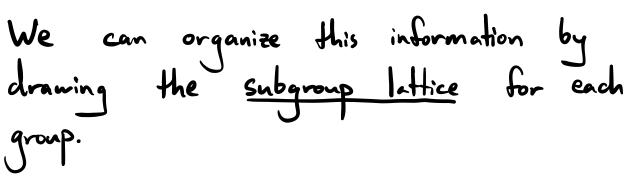
Subgroups Def: Let G be a group. A subgroup of G is a subset $H \subseteq G$ which is also a group under the same operation. Notation: H & G. If H≤G and H≠G, write H≤G. $\cdot \mathbb{Z} \leq \mathbb{Q} \leq \mathbb{R} \leq \mathbb{C}$ (as groups under +) Ex: • $\{1, -1\} \notin \mathbb{Q}^{\times} \notin \mathbb{R}^{\times} \notin \mathbb{C}^{\times}$ (as groups under ') T T T Q \ {0} R \ {0} C \ {0} • For any group G, the subset $\{e\}$ containing only the identity is a subgroup, called the trivial subgroup of G.

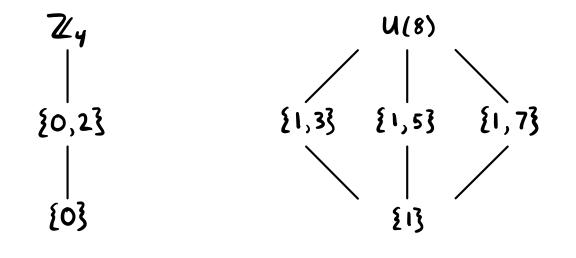
• 0 € 3Z 0=3(0) ✓

• 372 is closed under additive inverses -(3L) = 3(-L)

Therefore,
$$3\mathbb{Z} \leq \mathbb{Z}$$
.







Here, upward paths indicate inclusions.