HW1 Solutions

Section 2.1

15) $ty' + 2y = t^2 - t + 1 \quad y(1) = 1/2;\ t > 0$

We have $y' + \frac{2}{t}y = t - 1 + \frac{1}{t}$; so the integrating factor is

$$\mu(t) = e^{\int \frac{2}{t}dt} = t^2$$

so $t^2y' + 2ty = t^3 - t^2 + t$ and thus

$$\frac{d}{dt}\left[t^2 y\right] = t^3 - t^2 + t$$

$$t^2 y = \int t^3 - t^2 + t\,dt = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + C$$

so $C + \frac{1}{4} - \frac{1}{3} + \frac{1}{2} = \frac{1}{2}$. so $C = \frac{1}{12}$

and hence $y(t) = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{1}{12t^2}$.

Section 2.2

15) $y' = \frac{2x}{1+2y} \qquad y(2) = 0.$

a) We use separation of variables

$\int 1+2y\,dy = \int 2x\,dx$; so $y + y^2 = x^2 + C$

and hence $C = -4$.

Now we solve for y using the quadratic formula

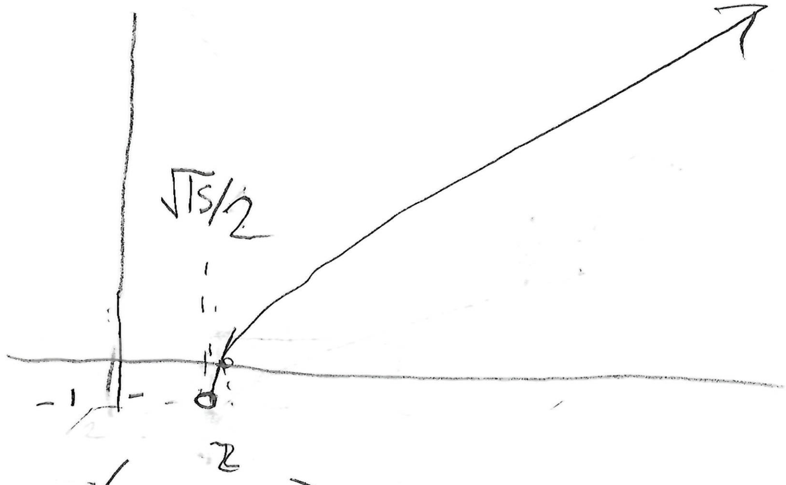
$$y = \frac{-1 \pm \sqrt{1 + 4(x^2 - 4)}}{2} = -\frac{1}{2} \pm \sqrt{x^2 - \frac{15}{4}}$$

We need 2 to belong to the domain

of y and $y(2) = 0$; hence

$y = -\frac{1}{2} + \sqrt{x^2 - \frac{15}{4}}$ with $x \in \left(\frac{\sqrt{15}}{2}, \infty\right)$

b)



c) $\left(\frac{\sqrt{15}}{2}, \infty\right)$

section 2.4

13| $y' = -4t/y$ $\quad y(0) = y_0$; $y_0 \neq 0$

We know that $\int y\,dy = \int -4t\,dt$

so $\frac{y^2}{2} = -2t^2 + c$ and hence $c = y_0^2/2$,

thus $y = \pm\sqrt{2}\sqrt{\frac{y_0^2}{2} - 2t^2}$

so if $y_0 > 0$; $y = \sqrt{y_0^2 - 4t^2}$; $|t| \lneq \frac{y_0}{2}$

if $y_0 < 0$; $y = -\sqrt{y_0^2 - 4t^2}$; $|t| \lneq \frac{|y_0|}{2}$