Homework 2 Solutions

Section 2.2

7| $\dfrac{dy}{dx} = \dfrac{x-e^{-x}}{y+e^y}$

$\int y + e^y \, dy = \int x - e^{-x} \, dx$

$y^2/2 + e^y = \dfrac{x^2}{2} + e^{-x} + C$

Section 2.4

12| $\dfrac{dy}{dt} = \dfrac{\cot(t)y}{1+y}$

$\dfrac{\partial}{\partial y}\left(\dfrac{dy}{dt}\right) = \dfrac{\cot(y)(1+y) - \cot(t)y}{(1+y)^2}$

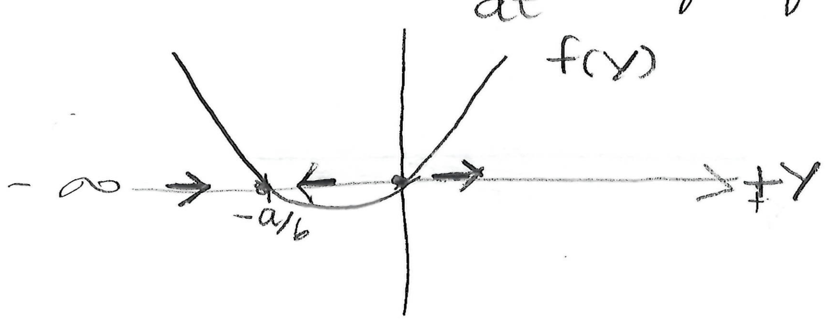
Therefore the condition on theorem 2.4.2 is satisfied by (t_0, Y_0) such that $y_0 \neq -1$ and $t_0 \neq \pi n$ for any integer n (or any $n \in \{0,-1,1,-2,2,\ldots\}$).

Section 2.5

2| First we will determine the equilibrium points. We have $0 = ay + by^2$; so

$y = \dfrac{-a \pm a}{2b} = 0, -\dfrac{a}{b}$.

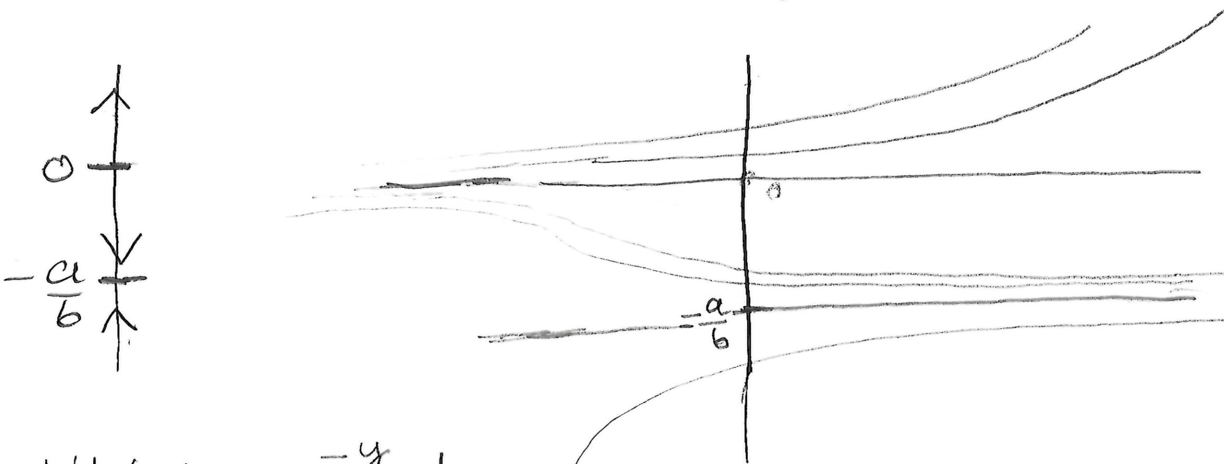
We also note that $\dfrac{dy}{dt} = ay + by^2$ so



so $y=0$ is unstable and $y=-\frac{a}{b}$ is stable.

Now we draw the phase line and sketch several graphs of solutions on the ty-plane:

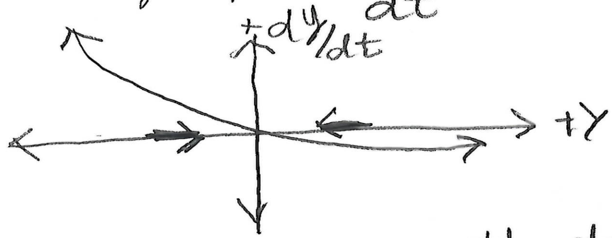
<u>Phase Line</u> <u>ty-plane</u>



5] $dy/dt = e^{-y} - 1$

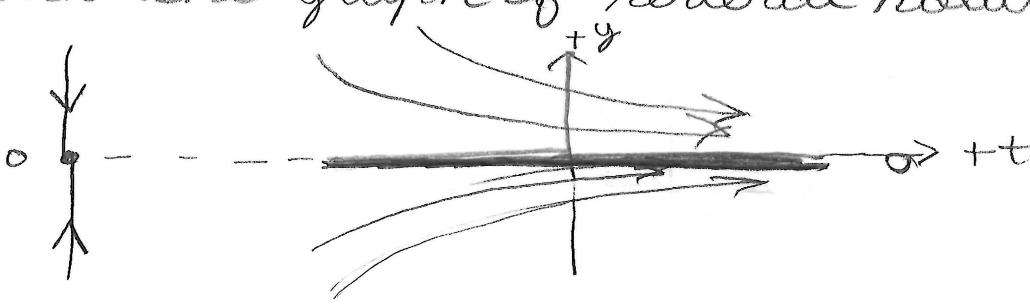
First we find the equilibrium points by making $e^{-y} = 1$. So the only equilibrium point is $y=0$.

Now we graph $\frac{dy}{dt} = e^{-y} - 1$



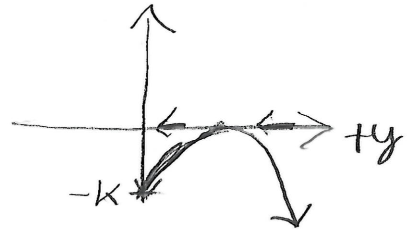
Hence we observe that $y=0$ is asymptotically stable.

Finally we draw the phase line and sketch the graph of several solutions



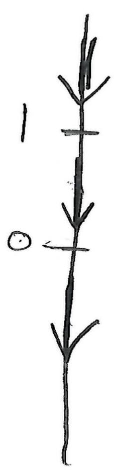
8| $\frac{dy}{dt} = -K(y-1)^2$; $K>0$.

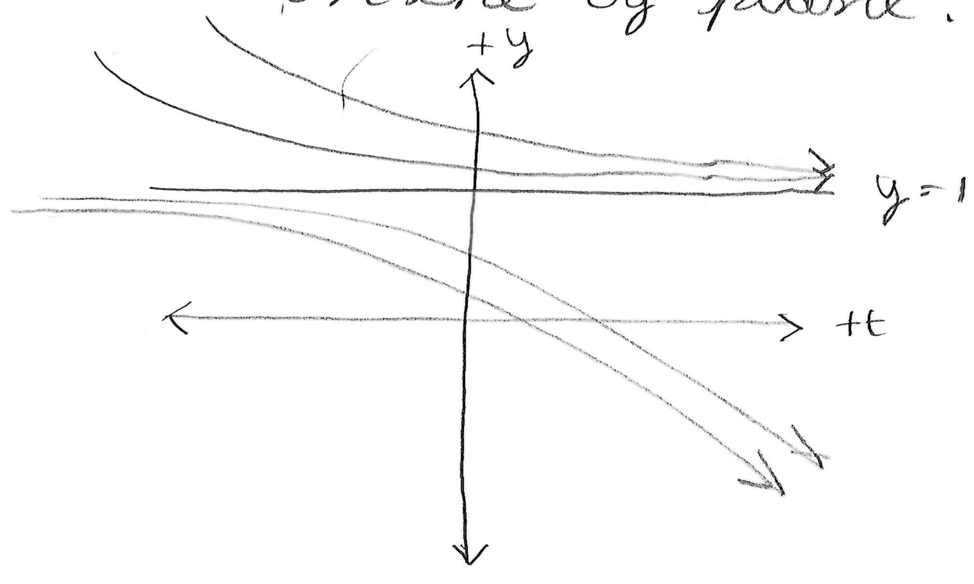
The only critical point of dy/dt is $y=1$. The following is a sketch of the graph of dy/dt



~~Iso~~ $y=1$ is semistable.

Now we sketch the phase line and several solutions on the ty-plane.

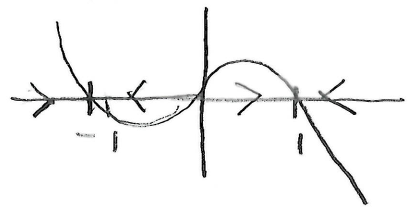




10) $\frac{dy}{dt} = y(1-y^2)$

The critical points are $y = 0, -1, 1$

a sketch of the graph of $\frac{dy}{dt}$ is



So $-1, 1$ are stable solutions and

0 is an unstable

Now we sketch the phase line and several solutions on the ty-plane

Phase line

ty-plane