# Solutions for HW3

## Section 2.6

5) $\frac{dy}{dx} = \frac{-ax+by}{bx+cy}$

First we write $(bx+cy)dy + (ax+by)dx = 0$ and note that $M(x,y) = ax+by$ and $N(x,y) = bx+cy$; so $M_y = b$ and $N_x = b$.
Therefore the equation is exact.
Now $\Psi(x,y) = \int ax+by\,dx = \frac{ax^2}{2} + byx + f(y)$
and thus $\Psi_y = bx + f'(y) = bx+cy$.
so $f(y) = \int cy\,dy = \frac{cy^2}{2} + d$
$\Psi(x,y) = \frac{ax^2}{2} + bxy + \frac{cy^2}{2} + d = 0$ is a solution for the differential equation. and hence

13) We have that $M(x,y) = 2x-y$ and $N(x,y) = 2y-x$; so $M_y = -1 = N_x$; and hence the differential equation is exact.
We have that $\Psi(x,y) = \int 2x-y\,dx = x^2 - yx + f(y)$
and hence $-x + f'(y) = 2y - x$. So
$0 = \Psi(x,y) = x^2 - yx + y^2 + d$.
The initial condition is $y(1) = 3$, so
$1 - 3 + 9 + d = 0$ and hence $d = -7$. So the solution to the initial value problem is given implicitly by $x^2 - yx + y^2 = 7$.

Now, by completing the square we have

$$\frac{3}{4}x^2 + \left(y - \frac{x}{2}\right)^2 = 7$$

so $y = \sqrt{7 - \frac{3x^2}{4}} + \frac{x}{2}$ (check the initial condition to discard - case,)

This means that the solution is valid as long as $3x^2 < 28$

18| We note that $M_y = 0$ and $N_x = 0$, so the separable equation is exact.

## Section 2.7

3| For practical reasons, we will first find the solution to the differential equation

$$y' = 0.5 - t + 2y \qquad y(0) = 1$$

For this we write

$y' - 2y = 0.5 - t$, so multiplying by the integration factor $e^{-2t}$ we get a linear equation; is so

$$e^{-2t}y = \int 0.5e^{-2t}dt - \int te^{-2t}dt = -\frac{0.5e^{-2t}}{2} + \frac{te^{-2t}}{2} + \frac{e^{-2t}}{4} + C;$$

where $C = 1$ and thus $y = \frac{t}{2} + e^{2t}$

Now we construct the following table to complete a, b, c, d.

| $t$ | $h=0.025$ | $h=0.05$ | $h=0.1$ | EXACT |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 0.025 | 1.0625 | | | |
| 0.05 | 1.1275 | 1.125 | | |
| 0.075 | 1.1951 | | | |
| 0.1 | 1.2655 | 1.26 | 1.25 | 1.2714 |
| 0.125 | 1.3388 | | | |
| 0.150 | 1.4151 | 1.406 | | |
| 0.175 | 1.4946 | | | |
| 0.2 | 1.5774 | 1.5641 | 1.54 | 1.5918 |
| 0.225 | 1.6638 | | | |
| 0.250 | 1.7539 | 1.7355 | | |
| 0.275 | 1.8478 | | | |
| 0.3 | 1.9458 | 1.9216 | 1.878 | 1.9721 |
| 0.325 | 2.0481 | | | |
| 0.350 | 2.1549 | 2.1237 | | |
| 0.375 | 2.2664 | | | |
| 0.4 | 2.3829 | 2.3436 | 2.2736 | 2.4255 |

## Section 2.8

2] $\dfrac{dy}{dt} = 1 - y^3; \quad y(-1) = 3$

We let $U = t+1$ and $V = Y - 3$; then we have $\dfrac{dV}{dU} = \dfrac{dV}{dt}\dfrac{dt}{dU} = \dfrac{dV}{dt}$. But $\dfrac{dV}{dt} = \dfrac{dy}{dt}$ by definition, so

$$\frac{dV}{dU} = 1 - (V-3)^3$$ with the initial condition when $U=0$ (so $t=-1$)

$V|_{U=0} = V|_{t=-1} = Y(-1) - 3 = 0$; so $V(0) = 0$.