# solutions to Homework set 4

**2.9** 5) Given $y_0$ we will compute the first few terms of the relation $y_{n+1} = 0.5\,y_n + 6$ to find a patern. Then we will use the well known factorization

$$(1-x)(1+x^2+\cdots+x^{k-1}) = (1-x^k).$$

First we have $y_0 = y_0$, $y_1 = \frac{y_0}{2} + 6$,

$$y_2 = \frac{y_1}{2} + 6 = \frac{y_0}{2^2} + \frac{6}{2} + 6$$

$$y_3 = \frac{y_0}{2^3} + \frac{6}{2^2} + \frac{6}{2} + 6$$

$$\vdots$$

$$y_k = \frac{y_0}{2^k} + \frac{6}{2^{k-1}} + \cdots + \frac{6}{2} + 6$$

so $y_n = \frac{y_0}{2^n} + 6\left(1 + \frac{1}{2} + \cdots + \frac{1}{2^{n-1}}\right) = \frac{y_0}{2^n} + 6\left(\frac{1-(1/2)^n}{1-1/2}\right)$

and hence $y_n = \frac{y_0}{2^n} + 12(1 - 1/2^n)$. Thus

$\lim_{n\to\infty} y_n = 12.$

**3.1** 9) $y'' + y' - 2y = 0 \qquad y(0) = 1 \quad y'(0) = 1$

The characteristic polynomial is

$r^2 + r - 2 = (r-1)(r+2)$ so $y_1 = ce^t$ and $y_2 = de^{-2t}$ are solutions to the diff. eq.

Now we have $y(0) = ce^0 + de^0 = c + d = 1$

and

$$y'(0) = ce^0 - 2de^0 = c - 2d = 1.$$

so $-3d = 0$ and hence $c = 1$. Therefore the solution to the initial value problem is $y = e^t$

(see graph at the end of solution.)

23| The characteristic polynomial is given by $r^2-(2\alpha-1)r+\alpha(\alpha-1)$ $=(r-\alpha)(r-(\alpha-1))$, so the solution of the diff. eq. is given by
$y=c_1e^{\alpha t}+c_2e^{(\alpha-1)t}$.
Note that depending on the initial value conditions, $c_1$ and $c_2$ might be 0. Hence if we want $\lim_{t\to\infty} y(t)=0$ regardless of $c_1$ and $c_2$ we must have $\alpha<0$. On the other hand if $c_1\neq 0$ or $c_2\neq 0$, $\lim_{t\to\infty}|y(t)|=\infty$ iff $\alpha>1$.

3.2| 9) We use Theorem 3.2.1 and consider

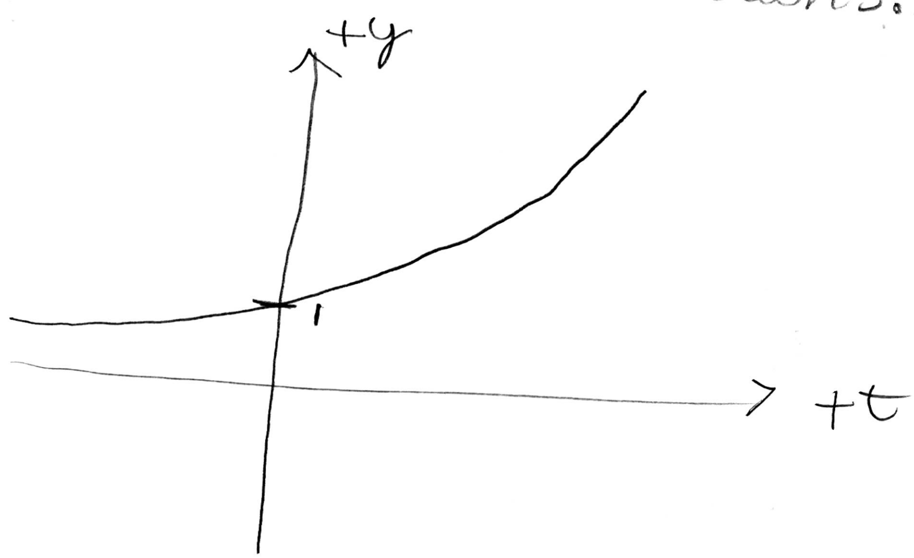
$$y''+\frac{3y'}{t-4}+\frac{4y}{t(t-4)}=2/t(t-4) \quad y(3)=0;\ y'(3)=-1$$

Here $p(t)=\frac{3}{t-4}$; $q(t)=\frac{4}{t(t-4)}$ and $g(t)=\frac{2}{t(t-4)}$; so the largest interval in which $p,q,g$ are continuous and has $t=3$ as an element is $(0,4)$. Thus by Theorem 3.2.1 the largest interval in which we can ensure that this problem has a unique solution is $(0,4)$.

24) $y'' + 4y = 0 \quad y_1(t) = \cos 2t$
$y_2(t) = \sin 2t$

First we check that $y_1, y_2$ are solutions, for this we note that $y_2'' = -4\sin 2t$ and $y_1'' = -4\cos 2t$; so clearly $y_1, y_2$ are solutions.
Now observe that $W(y_1, y_2)(t) = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix}$
$= 2\cos^2 2t + 2\sin^2 2t = 2$. So $y_1, y_2$ is a fundamental set of solutions.

Graph of $y = e^t$ (Section 3.1, problem 5)




note that as $t \to \infty$, $y \to \infty$