6 (ii) To show that E and $\overline{E}$ have the same set of limit pts.

① $x \in E' \Rightarrow \forall r, \ (N_r(x) \setminus \{x\}) \cap E \neq \phi$

$\Rightarrow \forall r, \ (N_r(x) \setminus \{x\}) \cap \overline{E} \neq \phi.$

$\Rightarrow x \in (\overline{E})'.$

② If $x \in \overline{E}'$, we need to show that $\forall r > 0, \ (N_r(x) \setminus \{x\}) \cap E \neq \phi.$

Let $r > 0$ be given

$x \in \overline{E}' \Rightarrow \exists y \in \overline{E} \cap (N_r(x) \setminus \{x\}).$

Case ① If $y \in E$, then $y \in (N_r(x) \setminus \{x\}) \cap E$ and $(N_r(x) \setminus \{x\}) \cap E \neq \phi$

Note
This does not say that $x \in E'$ right away.

Case ② If $y \notin E$, then $y \in E'$.

Take $r' = \min\{ r - d(x,y), d(x,y) \}$

$y \in E' \Rightarrow (N_{r'}(y) \setminus \{y\}) \cap E \neq \phi.$

By our choice of r', $(N_{r'}(y) \setminus \{y\}) \subset N_r(x) \setminus \{x\}.$

so $(N_r(x) \setminus \{x\}) \cap E \neq \phi.$

Here we have shown that for each $r > 0$, $(N_r(x) \setminus \{x\}) \cap E \neq \phi$

This implies $x \in E'$ #