Thm 2.20. If p is a limit pt. of a set E (in X), then every neighborhood of p contains infinitly many pts of E.

Pf. Suppose there is a neighborhood N_o of p that contains only a finite number of pts of E.

Let $N_o = N_{r_o}(p)$ for some $r_o > 0$

and let ~~A~~ $q_1, \ldots, q_k$ be those points in $N \cap E$ which are distinct from p, and put

$$r = \min\{r_o, d(p, q_1), d(p, q_2) \ldots, d(p, q_k)\}.$$

(Note: r is positive)

We claim that $N' = N_r(p)$ contains no points of E that is distinct from p.

① $N' = N_r(p) \subset N_{r_o}(p) = N_o$ (i.e. N' is a subset of N_o)

so any points of E in N' ~~is necessarily a point~~ that is distinct from p can only be one of $q_1 \ldots q_k$.

② $q_i \notin N'$ for all $i = 1 \ldots k$, as

$$d(q_i, p) \geq r \Rightarrow q_i \notin N_r(p).$$

Hence N' is a neighborhood of p that does not contain any points of E distint from p

i.e. p is not a limit pt. of E, a contradiction!

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