Math 2255, Spring 2017, Quiz 1, Jan 20 **Name:** ______________________________

1. [10] Determine the *order* of the given differential equations, and state whether the equation is *linear or nonlinear.*

$$\frac{d^2y}{dt^2} + \sin(t+y) = \sin tt, \qquad\qquad y''' + ty' + (\cos^2 t)y = t^3.$$

Answer: (2, nonlinear) , (3, linear).

2. [10] Find the solution of the given initial value problem.

$$y' - y = 1, \qquad y(0) = -2.$$

Answer: Using the method of integrating factor:

Solving $\frac{d \ln \mu(t)}{dt} = -1$, we take $\mu(t) = e^{-t}$. Next, multiply both sides by $\mu(t) = e^{-t}$:

$$\frac{d}{dt}\left[e^{-t}y\right] = e^{-t}\frac{dy}{dt} - e^{-t}y = e^{-t}$$

Integrate from 0 to t:

$$e^{-t}y - e^0 y(0) = -(e^{-t} - 1) = 1 - e^{-t}$$

Hence

$$y = e^t y(0) + e^t - 1 = -2e^t + e^t - 1 = -e^t - 1$$

(Don't forget to just plug-in the solution to check your answer.)

3. [10] Find the solution of the given initial value problem and state the maximum interval of definition determined by the given initial data,

$$\frac{dy}{dx} = -\frac{x}{2y}, \qquad y(0) = 1.$$

Answer: Rewrite the DE:

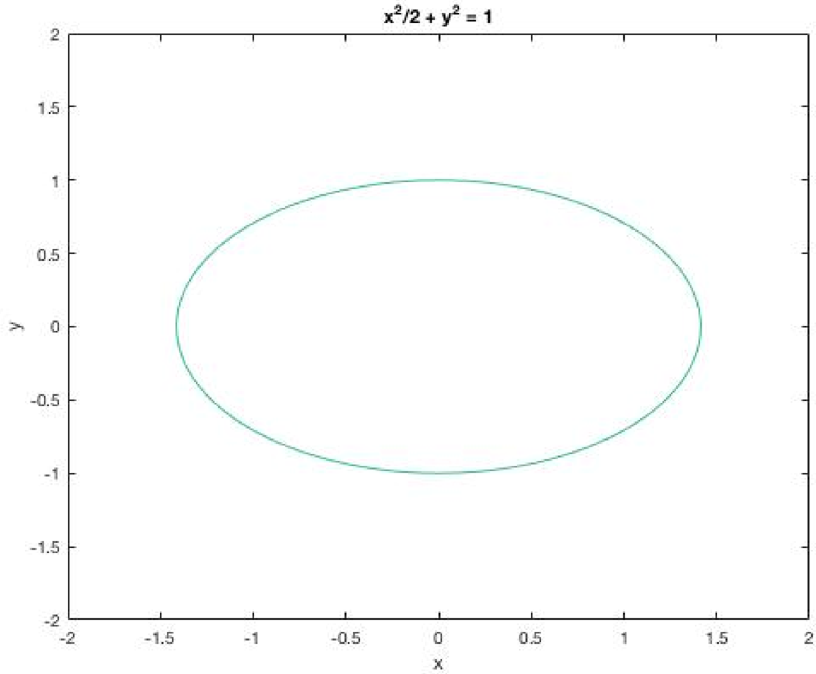
$$x + 2y\frac{dy}{dx} = 0$$

i.e. The DE is separable.

$$\frac{d}{dx}\left[\frac{x^2}{2} + y^2\right] = 0 \quad \Longrightarrow \quad \frac{x^2}{2} + y^2 = C.$$

To determine the constant C, plug in $(x_0, y_0) = (0, 1)$: $C = \frac{0}{2} + 1 = 1$. Hence the integral curve is given implicitly by

$$\frac{x^2}{2} + y^2 = 1.$$



The maximum interval of definition is $-\sqrt{2} < x < \sqrt{2}$ (the solution being the upper half of the ellipse).