Math 2255, Spring 2017, Quiz 3, Feb 13 **Name:** ______________________________

1. [10] Consider a linear, homogeneous, second order differential equation $y'' + p(t)y' + q(t)y = 0$ where $p(t), q(t)$ are continuous functions for all real number t. Which of the following is not true? No explanation is needed.

   (a) There is exactly one solution to the DE $y'' + p(t)y' + q(t)y = 0$ together with the initial condition $y(0) = 0$, $y'(0) = 0$.

   (b) There is at most one solution to the DE $y'' + p(t)y' + q(t)y = 0$ together with the initial condition $y(0) = 1$.

   (c) There are at least two solutions to the DE $y'' + p(t)y' + q(t)y = 0$ together with the initial condition $y(0) = 2$.

   (d) There are at least eleven solutions to the DE $y''+p(t)y'+q(t)y = 0$ together with the initial condition $y'(0) = 3$.

   Answer: (b) is false, since for any prescribed values $y(0) = 1$ and $y'(0) = y_0'$, the DE has a solution. So in fact the DE has infinitely many solutions if we only prescribe one initial condition of $y(0) = 1$

2. [10] Find the general solution of the DE $y'' - 4y' + \frac{25}{4}y = 0$.

   Answer: Solving the characteristic equation:

$$r^2 - 4r + \frac{25}{4} = 0, \quad \Longrightarrow \quad r_1 = 2 + \frac{3}{2}i, \quad r_2 = 2 - \frac{3}{2}i$$

   And the general solution is

$$y(t) = e^{2t}(c_1 \cos(\frac{3}{2}t) + c_2 \sin(\frac{3}{2}t))$$

3. [10] Solve the DE $y'' + 4y' + 3y = 0$, $y(0) = 4$, $y'(0) = -6$. Determine the behavior of solutions as t increases.

Solving the characteristic equation:

$$r^2 + 4r + 3 = (r+1)(r+3) = 0, \quad \Longrightarrow \quad r_1 = -1, \quad r_2 = -3.$$

The general solution is $y(t) = c_1 e^{-t} + c_2 e^{-3t}$. It remains to solve for c_1, and c_2. Compute

$$y'(t) = -c_1 e^{-t} - 3c_2 e^{-3t}.$$

Then

$$\begin{aligned} y(0) = 4 \qquad &\Longrightarrow \quad c_1 + c_2 = 4, \\ y'(0) = -6 \qquad &\Longrightarrow \quad -c_1 - 3c_2 = -6. \end{aligned}$$

Solving, we get $c_2 = 1$ and $c_1 = 3$. Hence

$$y(t) = 3e^{-t} + e^{-3t}.$$

As t increases, $y(t) \to 0$.