Math 2255, Spring 2017, Quiz 6, April 3 **Name:** TA

1. [5] In each of the following differential equations, determine intervals in which solutions are sure to exist.

$$y^{(4)} - e^t y'' + 10y = t,$$

all coeff are cont. in $(-\infty, \infty)$

$\Rightarrow$ solutions exist in $(-\infty, \infty)$.

$$ty''' + (\sin t)y'' + 3y = \cos t$$

$y''' + \frac{\sin t}{t} + \frac{3}{t} y = \frac{\cos t}{t}$.

coeff continuous except at $t=0$

$\Rightarrow$ solutions exist in $(-\infty, 0)$ or $(0, \infty)$

2. [10] Determine whether the given functions are linear dependent or independent. If they are linearly dependent, find a linear relation among them.

$$f_1(t) = 2t - 3, \quad f_2(t) = 2t^2 + 1, \quad f_3(t) = 3t^2 + t$$

Suppose $c_1 f_1(t) + c_2 f_2(t) + c_3 f_3(t) = 0$

$\Rightarrow c_1(2t-3) + c_2(2t^2+1) + c_3(3t^2+t) = 0$

$\Rightarrow (2c_2 + 3c_3)t^2 + (2c_1 + c_3)t + (-3c_1 + c_2) = 0$

$$\Rightarrow \begin{cases} 2c_2 + 3c_3 = 0 \\ 2c_1 + c_3 = 0 \\ -3c_1 + c_2 = 0 \end{cases} \Leftrightarrow \begin{cases} 2c_2 + 3c_3 = 0 \\ 6c_1 + 3c_3 = 0 \\ -6c_1 + 2c_2 = 0 \end{cases}$$

sum to be the first.

It does not imply that $c_1 = c_2 = c_3$, since, for example $c_2 = 3$, $c_3 = -2$, $c_1 = 1$

$\Rightarrow f_1(t) + 3f_2(t) - 2f_3(t) = 0$

$\Rightarrow$ they are linearly dependent.

3. (a)[5] Determine the indicated roots of the given complex number,

$$1^{\frac{1}{3}}$$

(b)[10] Find the solution of the given initial value problem.

$$y''' - y = 0; \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = -1,$$

(a) $1, \ -\frac{1}{2} + \frac{\sqrt{3}}{2} i, \ -\frac{1}{2} - \frac{\sqrt{3}}{2} i.$

(b) General sol: $y(t) = c_1 e^{t} + c_2 e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2} t + c_3 e^{-\frac{1}{2}t} \sin \frac{\sqrt{3}}{2} t.$

$y'(t) = c_1 e^{t} + \left(-\frac{c_2}{2} + \frac{\sqrt{3}}{2} c_3\right) e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2} t + \left(-\frac{\sqrt{3}}{2} c_2 - \frac{c_3}{2}\right) e^{-\frac{1}{2}t} \sin \frac{\sqrt{3}}{2} t$

$y''(0) = c_1 + \left(\frac{c_2}{4} - \frac{\sqrt{3}}{4} c_3\right) + \left(-\frac{3}{4} c_2 - \frac{\sqrt{3}}{4} c_3\right) = c_1 - \frac{c_2}{2} - \frac{\sqrt{3}}{2} c_3$

$y'(0) = c_1 - \frac{c_2}{2} + \frac{\sqrt{3}}{2} c_3, \quad y(0) = c_1 + c_2.$

$$\Rightarrow \begin{cases} c_1 + c_2 = 0 \\ c_1 - \frac{c_2}{2} + \frac{\sqrt{3}}{2} c_3 = 0 \\ c_1 - \frac{c_2}{2} - \frac{\sqrt{3}}{2} c_3 = -1. \end{cases} \Rightarrow \begin{array}{l} \sqrt{3} c_3 = 1 \Rightarrow c_3 = \frac{1}{\sqrt{3}}. \\ c_1 = -c_2 \\ -\frac{3}{2} c_2 + \frac{\sqrt{3}}{2} c_3 = 0 \Rightarrow c_2 = \frac{1}{\sqrt{3}} c_3 = \frac{1}{3}. \\ c_1 = -\frac{1}{3}. \end{array}$$

$$\Rightarrow y(t) = -\frac{1}{3} e^{t} + \frac{1}{3} e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2} t + \frac{1}{\sqrt{3}} e^{-\frac{1}{2}t} \sin \frac{\sqrt{3}}{2} t.$$