Courtesy of Prof Elliot Paquette

Name: ____________________

OSU

February 22, 2017

Worksheet

Problem 1. For each of the following problems, find a constant-coefficient, linear differential operator so that applying this operator to the function in question gives 0. For example, if we apply the operator $(D-5)$ to e^{5t} we get 0.

(a) $e^{3t}+e^{5t}$: *Hint: second–order.*

(b) $\sin(t)$: *Hint: one conclusion of Euler's formula is that* $\sin(t) = (e^{it} - e^{-it})/(2i)$. *You can use this identity and adapt what you got in the previous problem to this one.*

(c) $e^t + te^t$:

(d) $te^t\cos(t)$:

Problem 2. This problem gives another way of reaching a complex-valued differential equation for $\sin(t)$.

(a) Let $y(t) = \sin(3t)$. Compute the second derivative y''.

(b) Write a second–order, constant–coefficient, homogeneous differential equation involving y and y''.

(c) Factor this differential equation, i.e. write the equation you get in the previous part as $(D - r_1)(D - r_2)y = 0$ for some complex numbers r_1 and r_2.

Problem 3. The method of undetermined coefficients starts by creating a higher–order homogeneous differential equation from a lower–order inhomogeneous differential equation. This idea is only useful if the inhomogeneity is the solution of a simple differential equation itself. For example, if we want to solve

$$(D-1)(D-2)y = e^{5t}, \tag{0-1}$$

then using the observation from the example in the previous page, we see that

$$(D-5)(D-1)(D-2)y = 0.$$

The general solution of this higher–order equation, $C_1e^{5t} + C_2e^t + C_3e^{2t}$ is called the *form* of the solution in undetermined coefficients. In the following problems, determine the form.

(a) $(D-i)(D+i)y = te^t$:

(b) $y'' + 4y' + 4y = e^t \sin(3t) + \cos(t)$

Problem 4. Once you get the form, all those coefficients *except those corresponding to the solution of the associated homogeneous equation (in our example* (0-1)*, that would be* $C_2e^t + C_3e^{2t}$ *which is the general solution of the homogeneous equation* $(D-1)(D-2)y = 0$*)* are determined by the inhomogeneous equation (0-1). In the example from the previous problem, we can check that C_1 must be $1/12$ after pluggin C_1e^{5t} into the left hand side of (0-1). Hence the general solution of the inhomogeneous differential equation is

$$\frac{1}{12}e^{5t} + C_2e^t + C_3e^{2t}$$

In this problem, find the general solution of the inhomogeneous differential equation.

$$y'' + 6y' + 9y = \sin(t).$$

Problem 5. *Bonus* Find two linearly independent solutions of the second–order, linear, homogeneous equation

$$(D - p(t))(D - q(t))y = 0.$$

Expanding the left hand side, this is equivalent to

$$y'' - (p(t) + q(t))y' + (p(t)q(t) - q'(t))y = 0.$$

Hint: one solution comes from $(D - q(t))y = 0$*. For the second, use reduction of order.*