Belloni, M. [Belloni, Marino] (I-PARM); Ferone, V. (I-NAPL-AM); Kawohl, B. (D-KOLN)
Isoperimetric inequalities, Wulff shape and related questions for strongly nonlinear elliptic operators. (English summary)
Special issue dedicated to Lawrence E. Payne.

Summary: “We investigate the first eigenvalue of a highly nonlinear class of elliptic operators, which includes the $p$-Laplace operator $\Delta_p u = \sum_i \frac{\partial}{\partial x_i} (|\nabla u|^{p-2} \frac{\partial u}{\partial x_i})$, the pseudo-$p$-Laplace operator $\tilde{\Delta}_p u = \sum_i \frac{\partial}{\partial x_i} (|\frac{\partial u}{\partial x_i}|^{p-2} \frac{\partial u}{\partial x_i})$ and others. We derive the positivity of the first eigenfunction, simplicity of the first eigenvalue, and Faber-Krahn and Payne-Rayner type inequalities. We also address the question of symmetry for positive solutions to more general equations. Using a Pokhozhaev-type inequality and isoperimetric inequalities, as well as convex rearrangement methods, we generalize a symmetry result of S. Kesavan and F. Pacella [Appl. Anal. 54 (1994), no. 1-2, 27–37; MR1382205 (96m:35102)]. Our optimal domains are level sets of a convex function $H^0$. They have the so-called Wulff shape associated with $H$ and only in special cases are they Euclidean balls.”

© Copyright American Mathematical Society 2005, 2006