Belloni, M. [Belloni, Marino] (I-PARM); Kawohl, B. (D-KOLN)
A direct uniqueness proof for equations involving the \( p \)-Laplace operator. (English summary)


In this note the authors give a direct proof of the uniqueness of solutions for the problem

\[
\begin{cases}
\Delta_p u + \lambda |u|^{p-1} u = 0 \text{ in } \Omega, \\
u = 0 \text{ on } \partial\Omega,
\end{cases}
\]

where \( p \in (1, \infty) \), \( \Omega \subset \mathbb{R}^n \) is a bounded simply connected domain.

The proof is based on the observation that the functional

\[
J_p(v) = \int_{\Omega} |\nabla v|^p \, dx
\]

is concave in \( v^p \) for a nonnegative function \( v \).

This proof is generalized to a related result which states that a positive (weak) solution of

\[
\Delta_p u + f(x, u) = 0 \text{ in } \Omega, \\
u = 0 \text{ on } \partial\Omega
\]

is unique, provided \( f \) satisfies some hypotheses.

Reviewed by Jan Bochenek (Kraków)

**References**


9. Lindqvist, P.: On the equation $\nabla(\|\nabla u\|^{p-2}\nabla u) + \lambda\|u\|^{p-2}u = 0$, Proc. Amer. Math. Soc. 109, 157–164 (1990) MR1007505 (90h:35088)

10. Lindqvist, P.: Addendum to "On the equation $\nabla(\|\nabla u\|^{p-2}\nabla u) + \lambda\|u\|^{p-2}u = 0$$"$, Proc. Amer. Math. Soc. 116, 583–584 (1992) MR1139483


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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